ASSEMBLING of Elementary Theory Relativity

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ANNOTATION

List of principles and formulas belonging to the Elementary Theory of Relativity. The theory ratios of physical scales in non-moving frames of reference.

Content

1. Foreword

Two parallel inertial frames of reference are given in Newton's absolute space. The systems differ in the scales for measuring length, time, and mass. Angle in space are invariants. Absolute space is understood as a single frame of reference. All systems are equal until one of them is selected as a base (laboratory). The axes and origins of parallel systems continuously coincide in the space of a single reference system. In the laboratory (basic) frame of reference, the speed of light is a constant.

Any material point can be divided in half and placed in parallel reference systems. Between the points there is a rigid geometric connection in the transverse direction. Such material points are considered alternative. They constantly coincide in space and time of parallel reference systems. Points have their own units of measurement, length, time and mass in their own reference systems.

The rectangular coordinate system (x, y, z) is used to analyze the rectilinear motion of a point. The natural coordinate system (τ, n, b) is used to analyze the curvilinear motion of a point. Alternative material points make mutual translational motion.

Principles of elementary relativity:

1. Any physical processes written in dimensionless units (ratios) have the same values in all inertial frames of reference;

2.The speed of light is an invariant in the base (laboratory) frame of reference;

3. For alternative particles from neighboring reference systems, Newton's third law applies.

2. Elementary Theory of Relativity

Consequences of the principles of the Elementary Theory of Relativity:

1. The use of initial zero conditions for speed, length and time of motion makes it possible to obtain integral ratios of the parameters of neighboring reference systems without the use of differential analysis;

2. The external force impulse from the neighboring frame of reference must be balanced by the corresponding change in the momentum of the particle in its own frame of reference;

3. Numerical equality of relative forces in a closed mechanical system does not depend on the metric of proper reference systems.

Connection of intervals of length and time:

$$\Delta l_1 = \Delta l_0 \ K_l \ ;$$

$$\Delta t_1 = \Delta t_0 \ K_t \ ;$$

$$K_l \, K_t = 1 \, .$$

Here l_0, l_1 are the distances and time of movement of points in parallel (neighboring) reference systems from the origin. K_l, K_t - dimensionless coefficients of connection between length and time in parallel reference systems. The subscripts 0 and 1 mean belonging to their own reference systems.

Relationship between relative forces, force impulses and momentum:

$$F_{\tau 1} t_1 = m_0 v_0 \ ;$$

$$F_{\tau 0} t_0 = m_1 v_1 \ ;$$

$$\left|\vec{F}_{\tau 1}\right| = \left|-\vec{F}_{\tau 0}\right| \quad \text{(Newton's Third Law of Motion)}.$$

Here $m_0, m_1, v_0, v_1, F_{\tau 0}, F_{\tau 1}$ are masses, speeds and relative forces in their own reference frames.

Equilibrium conditions for alternative material points in space and time, basic ratios:

1. $l_1 t_1 \ = \ l_0 t_0 \ ;$
2. $m_1 l_1 \ = \ m_0 l_0 \ ;$
3. $m_0 t_1 \ = \ m_1 t_0 \ ;$
4. $\frac{m_1^2 l_1}{t_1} = \frac{m_0^2 l_0}{t_0} \ ;$
5. $m_1^2 v_1 = m_0^2 v_0 \ .$
6. $\omega_1 t_1 = \omega_0 t_0$

Generalized relative speed:

$$\triangle = \frac{l_1}{t_0} = \frac{l_0}{t_1} \ ;$$

$$\triangle^2 = v_0 v_1 \ ;$$

$$\vec{\triangle} \perp \vec{v}_1 , \vec{v}_0 \ .$$

An equilateral triangle is the designation of relative speed in texts and formulas.

3. VERSION OF Elementary Relativity

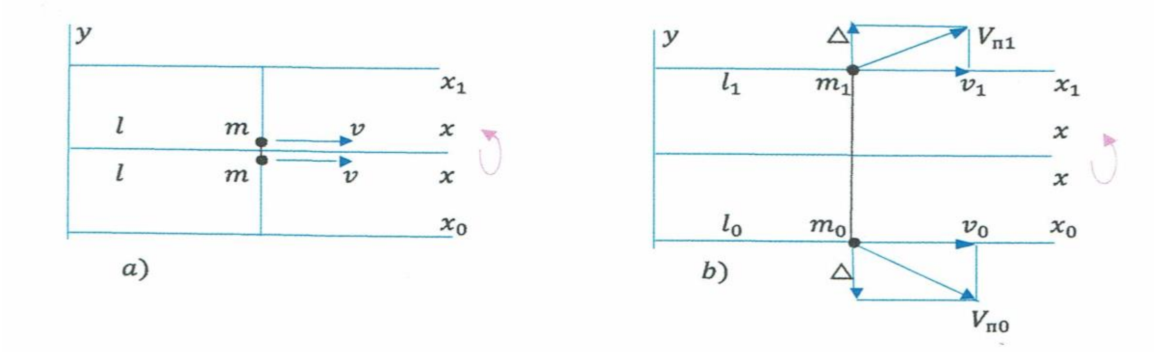
3.1) Recording of linear pulses for equal reference systems:

$$V_{п1}^2 m_1^2 = v_1^2 m_1^2 + \triangle^2 \ m_1^2;$$

$$V_{п0}^2 m_0^2 = v_0^2 m_0^2 + \triangle^2 \ m_0^2;$$

$$v_1^2 m_1^2 + \triangle^2 \ m_1^2 = v_0^2 m_0^2 + \triangle^2 \ m_0^2.$$

Here $V_{п0}, V_{п1}$ are the total speeds of movement of points along the trajectory, taking into account the transverse rotation.



a) Alternative points in absolute reference system.

b) Alternative points in parallel reference systems.

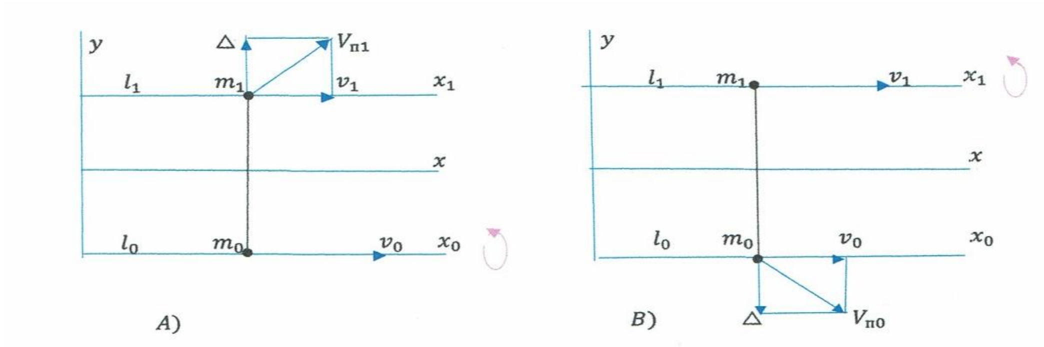
3.2) Recording the squares of linear impulses for unequal reference systems:

$A)\quad V_{п1}^2 m_1^2 = v_1^2 m_1^2 + \triangle^2\, m_1^2;$

$\triangle^2\, m_1^2 = v_0^2 m_0^2.$

$B)\quad V_{п0}^2 m_0^2 = v_0^2 m_0^2 + \triangle^2\, m_0^2;$

$\triangle^2\, m_0^2 = v_1^2 m_1^2.$



A) The reference system with the subscript 0 is taken as the base (laboratory). The neighboring system is considered parallel.

B) The reference system with the subscript 1 is taken as the base (laboratory). The neighboring system is considered parallel.

3.3) Particular solution of the system of equations for variant A).

In the basic (laboratory) frame of reference, the speed of light is a constant, $c_0 = c = 30 \cdot 10^7$ m/s m/s. The solution is sought in the form of the relation v_1/v_0 . The equations are reduced to the following form:

$$m_1^2 c^2 = v_1^2 m_1^2 + \triangle^2\, m_1^2\ ;$$

$$m_1^2 c^2 = v_0^2 m_0^2\ .$$

Correlation of distances, time, mass and speed of movement:

$$m_1 = \frac{m_0}{\sqrt{1-\frac{\triangle^2}{c^2}}};$$

$$l_1 = l_0\sqrt{1-\frac{\triangle^2}{c^2}}\ ;$$

$$t_1 = \frac{t_0}{\sqrt{1-\frac{\triangle^2}{c^2}}}\ ;$$

$$v_1 = v_0\left(1-\frac{\triangle^2}{c^2}\right);$$

$$V_{п1}^2 = {v_1}^2 + \triangle^2 .$$

Generalized relative speed:

$$\frac{l_1}{t_0} = \frac{l_0}{t_1} = \triangle = v_0\sqrt{1-\frac{\triangle^2}{c^2}}\ ;$$

$$\triangle_1^2 = \frac{v_0^2 c^2}{c^2+v_0^2} = \frac{v_0^2}{1+\frac{v_0^2}{c^2}} .$$

Here $\triangle_1$ is the calculated relative rotation speed in a parallel frame of reference. In the base frame of reference, the relative velocity is not defined.

The calculated relative speed changes the form of the relationship:

$$m_1 = m_0/\sqrt{1-\frac{\triangle_1^2}{c^2}} = m_0\sqrt{1+\frac{v_0^2}{c^2}}\ ;$$

$$l_1 = l_0\sqrt{1-\frac{\triangle_1^2}{c^2}} = l_0/\sqrt{1+\frac{v_0^2}{c^2}};$$

$$t_1 = t_0/\sqrt{1-\frac{\triangle_1^2}{c^2}} = t_0\sqrt{1+\frac{v_0^2}{c^2}};$$

$$c_1 = c/\left(1+\frac{v_0^2}{c^2}\right)\ ; \qquad \text{(from } m_1^2 c_1 = m_0^2 \text{c)}$$

$$v_1 = v_0\left(1-\frac{\triangle_1^2}{c^2}\right) = \frac{v_0}{1+\frac{v_0^2}{c^2}};\ \text{при/at} \begin{cases} v_0 = 0,\ v_1 = 0 \\ v_0 \ll c\ ,\ v_1 \approx v_0 \\ v_0 = c,\ v_1 = \frac{c}{2} \end{cases}$$

$$V_{п1}^2 = v_1^2 + \triangle_1^2 = \frac{v_0^2}{\left(1+\frac{v_0^2}{c^2}\right)^2} + \frac{v_0^2}{1+\frac{v_0^2}{c^2}};\ \text{at} \begin{cases} v_0^2 = 0,\ V_{п1}^2 = 0 \\ v_0^2 \ll c^2,\ V_{п1}^2 \cong 2v_0^2 \\ v_0^2 = c^2,\ V_{п1}^2 = \frac{3}{4}\text{c}^2 \end{cases}$$

For each value v_0 in the base frame of reference there is its own parallel frame of reference.

3.4) Dynamic equations for option A), initial zero conditions for speed and time:

$$F_{\tau 1}t_1 = m_0 v_0 = m_1 \triangle_1 ;$$

$$\left|\vec{F}_{\tau 1}\right| = \left|-\vec{F}_{\tau 0}\right| ;$$ (Newton's Third Law of Motion)

$$F'_{\tau 0}t_0 = m_0 v_0 ;$$

$$\left|\vec{F}'_{\tau 0}\right| \neq \left|\vec{F}_{\tau 0}\right| .$$

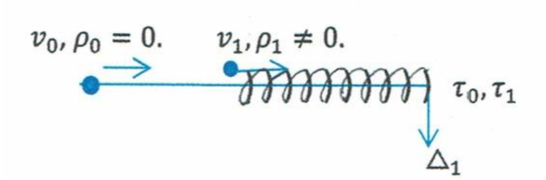
Here $F'_{\tau 0}$ is Newton's external force in the base frame of reference along the motion trajectory. The last system of equations is reduced to the form:

$$F_{\tau 0}t_0 = \frac{m_0 v_0}{\sqrt{1+\frac{v_0^2}{c^2}}} ;$$

$$F'_{\tau 0}t_0 = m_0 v_0 ;$$

$$(F'_{\tau 0} + F_{\tau 0})t_0 = m_0 v_0 + \frac{m_0 v_0}{\sqrt{1+\frac{v_0^2}{c^2}}} .$$

Two force impulses act on a material point in the base frame of reference. Newton's classical impulse and relativistic impulse from a parallel frame of reference. It can be assumed that the relativistic momentum is a linear analogue of the intrinsic torsion of a point. The sum of impulses is equal to the generalized mechanical impulse along the trajectory of motion. The movement of the point occurs along a helix with an infinitely small radius of rotation.



3.5) The form of writing the generalized linear momentum of a free particle in the laboratory frame of reference (subscripts are cancelled):

$$p = mv + \frac{mv}{\sqrt{1+\frac{v^2}{c^2}}} .$$

The resulting force acting on the particle:

$$F = \frac{dp}{dt} = m\frac{dv}{dt} + m\frac{dv}{dt}\left(1+\frac{v^2}{c^2}\right)^{-\frac{3}{2}} .$$

The resulting acceleration acting on the particle:

$$a = a_\tau + a_\tau\left(1+\frac{v^2}{c^2}\right)^{-\frac{3}{2}} .$$

Total kinetic energy of a relativistic particle:

$$E_k = \frac{mv^2}{2} + mc^2 - \frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}} = E_s + E_r .$$

Energy - Momentum relation:

$$E_r = mc^2 - \frac{c^2}{v} p_r \;;$$

$$E_r^2 = 2E_r mc^2 - \; p_r^2 c^2 \;.$$

Here:

$E_r = (E_k - \frac{mv^2}{2}) \;-\;$ relativistic component of the total kinetic energy of a point;

$p_r = (p - mv) -\;$ relativistic component of the total mechanical momentum;

$mc^2 \;-\;$ self-energy of a material point;

$p_s -$ Newton's classical mechanical momentum.

Effective mass of a material point:

$$M = \; m + m / \sqrt{1 + \frac{v^2}{c^2}} \;.$$

For massless particles of the photon type, there are no external forces. Ratios apply:

$$E_r = \frac{c^2}{v} |p_s - p_r| \;\Longrightarrow\; E_\gamma = c \cdot p_\gamma \;; \text{(electromagnetic pulse)}.$$

A photon cannot come from nowhere, there must be a body or a system of bodies that emits it. After emission, the photon forgets its source and exists on its own without mass.

4. Lorentz transformations

For events separated in space and time, Lorentz transformations follow from the analysis of cause-and-effect relationships. An example of such events is the beginning of the movement of points alternative in mass from different centers of coordinates at different times. Direct and inverse transformations of coordinates are connected by mathematical formalism and do not explain the physical difference between the events taking place.

One-dimensional transformations of coordinates and time lead to results on the scales of the base and parallel reference systems. The derivation of formulas based on clock synchronization at various points in space is given in a separate article.

Direct transformations. The position of the point $\boldsymbol{m_1}$ in the base frame of reference through the coordinates of the parallel frame of reference.	**Reverse transformations**. The position of the point $\boldsymbol{m_0}$ in the parallel frame of reference through the coordinates of the base frame of reference.

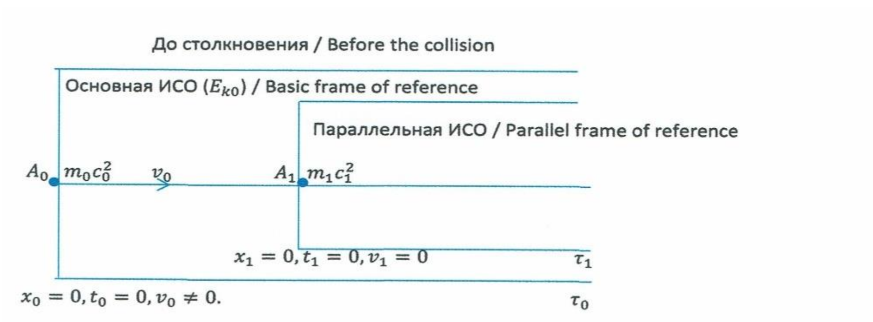
$x_0 = x_1\left(1+\frac{v_0^2}{c^2}\right)^{\frac{1}{2}} + v_0 t_1\left(1+\frac{v_0^2}{c^2}\right)^{-\frac{1}{2}}$	$x_1 = \frac{x_0 - v_0 t_0}{\left(1+\frac{v_0^2}{c^2}\right)^{1/2}} \frac{1}{\left(1-\frac{v_0^2}{c^2}\right)}$
$t_0 = t_1\left(1+\frac{v_0^2}{c^2}\right)^{-\frac{1}{2}} + x_1\frac{v_0}{c^2}\left(1+\frac{v_0^2}{c^2}\right)^{\frac{1}{2}}$	$t_1 = \frac{t_0 - x_0\frac{v_0}{c^2}}{\left(1+\frac{v_0^2}{c^2}\right)^{-1/2}} \frac{1}{\left(1-\frac{v_0^2}{c^2}\right)}$
$y_0 = y_1 = 0$	$y_1 = y_0 = 0$
$z_0 = z_1 = 0$	$z_1 = z_0 = 0$

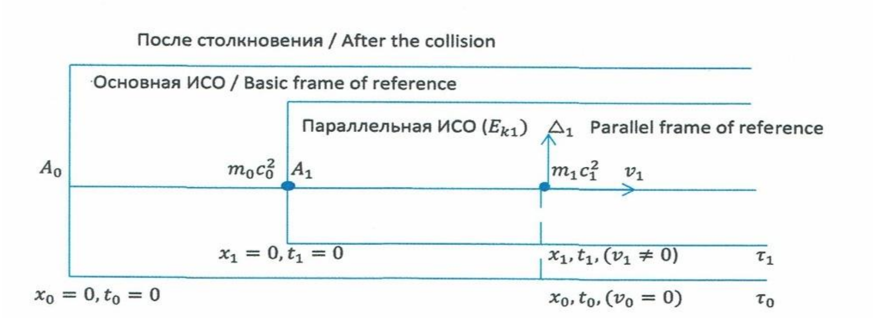
Energy Exchange.

Point $\mathbf{m_0}$ knocks out point $\mathbf{m_1}$ from the center of coordinates $\mathbf{A_1}$, stops and replaces it in a parallel frame of reference. Its coordinates in the parallel reference system have zero values and zero linear velocity. The rest energy of the point $\mathbf{m_0 c^2}$ in the main frame of reference remains unchanged. The replaces process corresponds to the inverse Lorentz transformation.

Point $\mathbf{m_1}$ after the impact acquires its own linear mechanical momentum in a parallel frame of reference. The rest energy of the point $\mathbf{m_1 c_1^2}$ remains unchanged. A point moves simultaneously in two frames of reference. The connection of coordinates and time between two frame of reference corresponds to direct Lorentz transformations. Elastic collision of material points is considered.

Energy exchange illustration.





When material points collide at the junction of coordinates, the parallel reference system tries to replace the base system. There is a conflict of reference systems at relativistic speeds.

The parameters of the basic reference system begin to depend on the parameters of the parallel system.

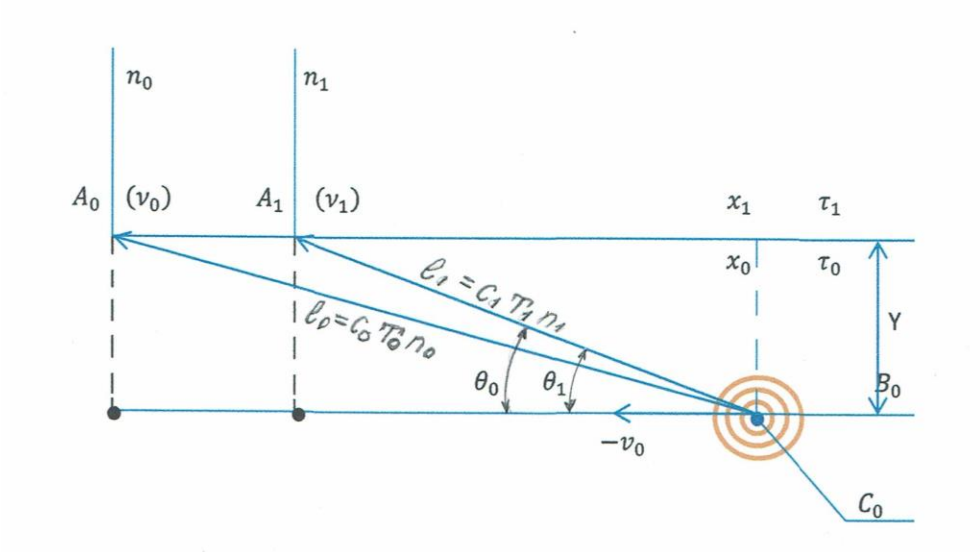
Connection of energies in alternative reference systems:

$$\frac{v_0}{c_0} = \frac{v_1}{c_1}; \quad m_0^2 v_0 = m_1^2 v_1;$$

$$E_{k1} = E_{k0}\left(1 + \frac{v_0^2}{c^2}\right)^{-\frac{3}{2}}.$$

5. Relativistic Doppler effect

Two parallel reference systems and a source of electromagnetic signals of constant frequency are given in Newton's absolute space. One of the parallel reference systems is taken as the base, the second remains parallel. The source moves in a vacuum, the source velocity is known only in the basic frame of reference. Signal receivers are located in the centers of coordinates of parallel systems ($\mathbf{A_0}, \mathbf{A_1}$). It is required to find the ratios of the frequencies of emission and reception of a signal in different frames of reference.



Here:

$l_0 = c_0 T_0 n_0 = \lambda_0 n_0$;

$l_1 = c_1 T_1 n_1 = \lambda_1 n_1$;

$c_0 = c \approx 30 * 10^7 \ {}^{m}/_{s} -$ the speed of light in the base frame of reference;

$c_1 = c/\left(1 + \frac{v_0^2}{c^2}\right) -$ the speed of light in the parallel frame of reference;

$T_0, T_1 -$ periods of received signals;

$\lambda_0, \lambda_1 -$ wavelengths of received signals.

By virtue of the basic principle of relativity:

$$\frac{l_0}{\lambda_0} = \frac{l_1}{\lambda_1} = n_0 = n_1 = n.$$

Here, $\boldsymbol{n} -$ is the number of oscillation cycles of signal receivers from a series of natural numbers 1, 2, 3, 4, 5, ..., accept $n = 1$.

The radiation source is in one of the parallel reference systems. We apply direct and inverse transformations of the Lorentz coordinates. We find the frequency ratios for two options for the location of the signal source.

$$\nu_1 = \nu_0 \frac{1-\beta\cos\theta_1}{\sqrt{1+\beta^2}}; \qquad \nu_1 = \nu_0 \frac{1-\beta^2}{\sqrt{1+\beta^2}(1+\beta\cos\theta_0)}.$$

Here, $\beta = v_0/c$.

Let's combine the centers of coordinates in one point. Then, $\cos\theta_0 = \cos\theta_1 = \cos\theta$. Let us denote the ratio of frequencies, $\boldsymbol{\nu_1/\nu_0 = \gamma}$. We exclude the common cosine from the equations. We obtain the general equation for the longitudinal Doppler effect. The equation has four inversion solutions:

$$\gamma^2(1+\beta^2) - 2\gamma\sqrt{1+\beta^2} + (1-\beta^2) = 0.$$

Solutions of the equation at a constant value ν_0 :

1) $\gamma = \frac{1-\beta}{\sqrt{1+\beta^2}} \to \nu_1 = \nu_0 \frac{1-\beta}{\sqrt{1+\beta^2}}$;

2) $\gamma = \frac{1+\beta}{\sqrt{1+\beta^2}} \to \nu_1' = \nu_0 \frac{1+\beta}{\sqrt{1+\beta^2}}$.

Solutions of the equation at a constant value ν_1 :

3) $\gamma^{-1} = \frac{\sqrt{1+\beta^2}}{1-\beta} \to \nu_0 = \nu_1 \frac{\sqrt{1+\beta^2}}{1-\beta}$;

4) $\gamma^{-1} = \frac{\sqrt{1+\beta^2}}{1+\beta} \to \nu_0' = \nu_1 \frac{\sqrt{1+\beta^2}}{1+\beta}$.

For the **parallel** reference system: $\nu_1' -$ signal source frequency; $\nu_1 -$ signal reception frequency, β - changes sign:

$\nu_1 = \nu_1' \frac{1-\beta}{1+\beta}$ − frequency redshift;

$\nu_1 = \nu_1' \frac{1+\beta}{1-\beta}$ − frequency blue shift.

For the **basic** reference system: $\nu_0' -$ signal source frequency; $\nu_0 -$ signal reception frequency, β- changes sign:

$\nu_0 = \nu_0' \frac{1+\beta}{1-\beta}$ − frequency blue shift;

$$\nu_0 = \nu_0' \frac{1-\beta}{1+\beta} -$$ frequency redshift.

There are two transverse Doppler effects. The signal source constantly rotates around a common center of coordinates. The signal receiver belongs to either center A_0 or center A_1. Accordingly, the emitter is located in the opposite frame of reference:

$$\nu_0 = \nu_1 \frac{\sqrt{1+\beta^2}}{1-\beta^2} \; ; \qquad \nu_0 \in \; A_0 \; ;$$

$$\nu_1 = \nu_0 \frac{1}{\sqrt{1+\beta^2}} \; ; \qquad \nu_1 \in \; A_1 \; .$$

From the expansion of functions in Taylor series, we can state that for $|\beta| \ll 1$:

$$\sqrt{1+\beta^2}^{\,-1} \cong \sqrt{1-\beta^2}^{\,+1} .$$

The last formulas are reduced to the form of the transverse Roemer effect:

$$\nu_0 \cong \nu_1 (1-\beta^2)^{-\frac{3}{2}} \; ; \qquad \nu_1 \cong \nu_0 \sqrt{1-\beta^2} .$$

6. Bremsstrahlung and quantum mechanics of the electron

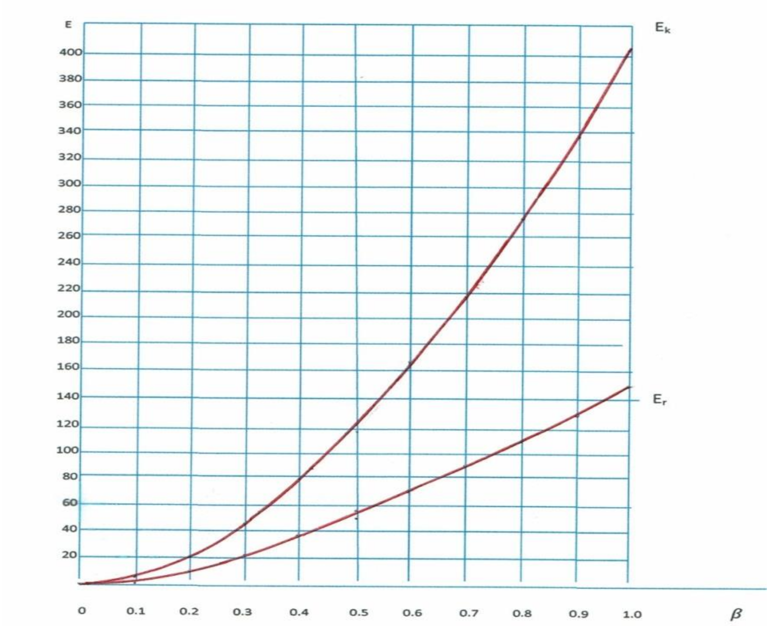
Continuous change in electron energy without emission of photons:

$$E_{ki} = \frac{m_e v_i^2}{2} + m_e c^2 - \frac{m_e c^2}{\sqrt{1+\frac{v_i^2}{c^2}}} = E_{si} + E_{ri} \; .$$

Here: m_e – The mass of an electron, $9{,}1 * 10^{-31}$ $\langle kg \rangle$;

v – Speed of free electrons in vacuum $\langle m/s \rangle$;

c – Speed of light in vacuum $30 * 10^7$, $\langle m/s \rangle$.



Elementary quantum of relativistic energy for an electron:

$$E_{ri} = m_e c^2 - \frac{m_e c^2}{\sqrt{1+\frac{v_i^2}{c^2}}} = h\nu_i$$

Quantum energy reduction of an electron in a bremsstrahlung electric field:

$$E_{si} = E_{ki} - E_{ri}\ ;$$

The first quantum of electromagnetic energy (photon) of a certain frequency is emitted. There is a subsequent restoration of the form of recording the total kinetic energy:

$$E_{k,(i+1)} = E_{si}\ ;$$

$$v_{i+1} < v_i;$$

$$\frac{m_e v_i^2}{2} = \frac{m_e v_{i+1}^2}{2} + m_e c^2 - \frac{m_e c^2}{\sqrt{1+\frac{v_{i+1}^2}{c^2}}}.$$

The total energy of the electron motion after recombination is equal to the classical kinetic component of the energy before recombination.

Energy recombination leads to equations in dimensionless units:

$$x_{i+1}^3 + x_{i+1}^2(5-2x_i) + x_{i+1}\left(8-6x_i+x_i^2\right) - \left(4x_i - x_i^2\right) = 0.$$

Here, $x_i = \frac{v_i^2}{c^2}$; $x_{i+1} = \frac{v_{i+1}^2}{c^2}$ – squares of the ratio of speeds.

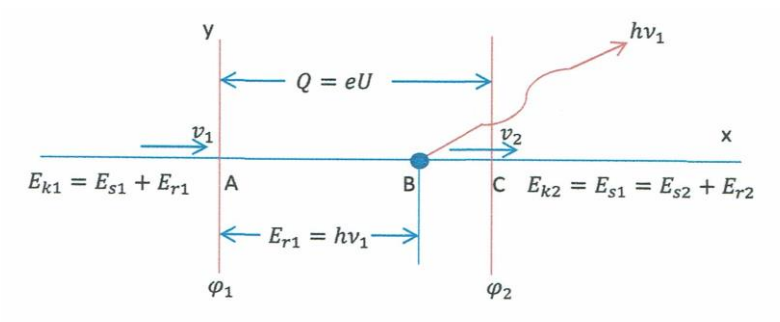
Assuming that the initial velocity of an electron is $v_1 \cong c$ and $x_1 = 1$, the first recombination equation takes the form:

$$x_2^3 + 3x_2^2 + 3x_2 - 3 = 0.$$

Equation roots: $x_1 = 0{,}59$; $x_1 = -1{,}79 + 1{,}37i$; $x_1 = -1{,}79 - 1{,}37i$.

Linear velocity of an electron after the first recombination:

$$\frac{v_2^2}{c^2} = 0{,}59; \quad \frac{v_2}{c} = 0{,}7681; \quad v_2 \approx 23{,}0 * 10^7\ m/s.$$



Here: Q – barrier energy width, $\langle eV \rangle$;

$h -$ Planck's constant, $6{,}626 * 10^{-34}$ $\langle J * s \rangle$;

$e -$ charge of an electron, $1{,}602 * 10^{-19}$ $\langle C \rangle$;

$U -$ electric potential difference at the barrier boundaries, $\langle V \rangle$;

$\nu_1 -$ radiation frequency $\langle Hz \rangle$.

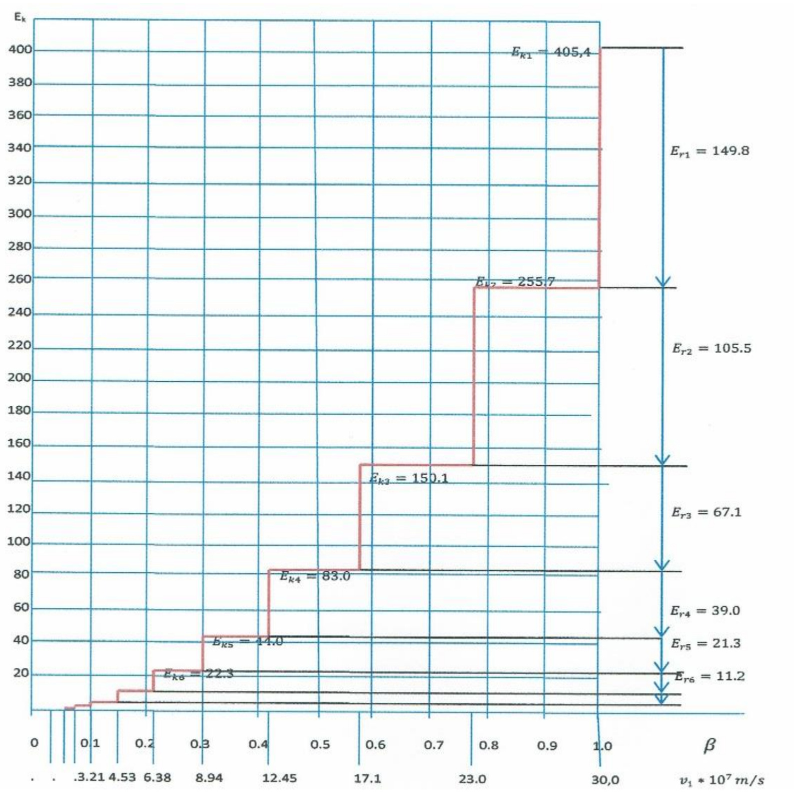
The total kinetic energy consists of the sum of relativistic quanta:

$$E_k = \sum E_{ri} + E_{k\,min}; \quad (E_{k\,min} \ll E_k)$$

$E_{k\,min} = \frac{mv_{min}^2}{2}$, the minimum portion of kinetic energy in the classical form.

Below is the calculation of parameters for the first 11 steps of energy recombination.

v_i * $10^7 m/s$	$\beta_i = \frac{v_i}{c}$	$x_i = \left(\frac{v_i}{c}\right)^2$	E_{ki} keV	$E_{ri} = h\nu_i$ keV	ν_i $* 10^{19}\ Hz$	λ_i * $10^{-3}\ nm$	T_i $* 10^{-19} s$
v_1 30.00	1.0000	1,000000	405,4	149,80	3,62	8,29	0,276
v_2 23.00	0,7664	0,587401	255,7	105,50	2,55	11,76	0,392
v_3 17.10	0,5700	0,324934	150,1	67,10	1,62	18,49	0,617
v_4 12.45	0,4160	0,172199	83,0	39,00	0,94	31,91	1,064
v_5 8.94	0,2981	0,088857	44,0	21,30	0,51	58,29	1,923
v_6 6.38	0,2125	0,045166	22,3	11,20	0,27	111,1	3,704
v_7 4.53	0,1509	0,022774	11,6	5,72	0,138	217,4	7,246
v_8 3.21	0,1069	0,011436	5,81	2,90	0,070	428,0	14,286
v_9 2.27	0,0756	0,005730	2,92	1,46	0,035	857,0	28,571
v_{10} 1.61	0,0536	0,002868	1,47	0,735	0,0177	1695,0	56,500
v_{11} 1.14	0,0379	0,001435	1,105	0,368	0,00834	3597,1	119,904



In the quantum regime, having given up the last possible photon of radiation, the electron abruptly loses its relativistic properties. That minimum amount of kinetic energy must remain, which is necessary for the return of an electron to the stationary orbit of an atom of matter.

$$E_{rmin} = m_e c^2 - \frac{m_e c^2}{\sqrt{1+\frac{v_{min}^2}{c^2}}} = h\nu_{min}$$

Here:

E_{rmin} – minimum quantum of energy;

v_{min} – minimum recombination speed.

Recording the generalized linear momentum of a free electron:

$$p = m_e v + \frac{m_e v}{\sqrt{1+\frac{v^2}{c^2}}}.$$

After each act of energy recombination, the law of conservation of the record of the total geometric momentum of the system in vector form must be fulfilled:

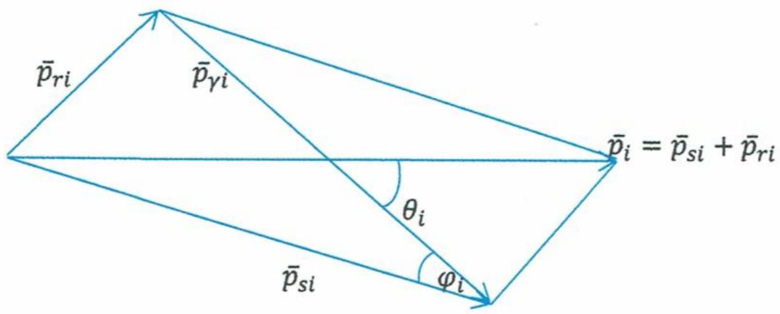
$$\vec{p}_i = \vec{p}_{si} + \vec{p}_{ri} ;$$

$$\vec{p}_{\gamma i} = \vec{p}_{si} - \vec{p}_{ri} .$$

Here:

$\vec{p}_i$ – the vector of the total geometric momentum of the electron;

$\vec{p}_{\gamma i}$ – the vector of the electromagnetic impulse of the bremsstrahlung photon.



Recording the momentum ratios for an arbitrary energy quantum:

$$p_i p_{\gamma i} \cos\theta_i = p_{si}^2 - p_{ri}^2 ;$$

$$p_{si} = m_e v_i ;$$

$$p_i^2 = 2p_{si}^2 + 2p_{ri}^2 - p_{\gamma i}^2 ;$$

$$p_{\gamma i} = \frac{c}{v_i}(p_{si} - p_{ri}) ;$$

$$p_{ri} = \frac{m_e v_i}{\sqrt{1+\frac{v_i^2}{c^2}}} ;$$

$$\frac{h\nu_i}{c} = p_{\gamma i}.$$

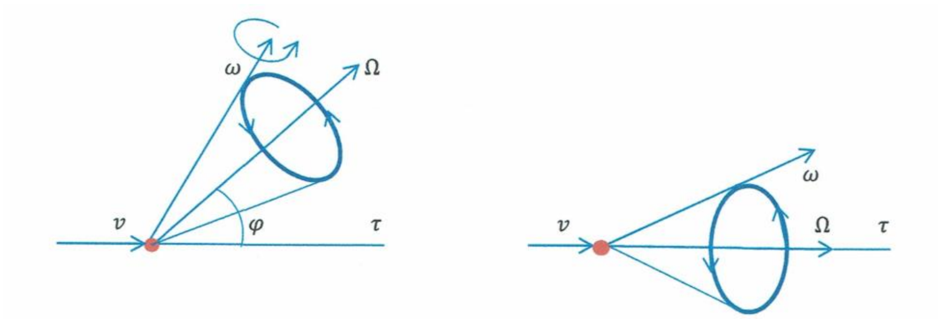
The actual angle of observation of photon radiation is determined by the expression:

$$\cos\varphi_i = \frac{p_{si}^2 - p_{ri}^2 + p_{\gamma i}^2}{2p_{si} * p_{\gamma i}} = \cos(\vec{p}_{si}; \vec{p}_{\gamma i}).$$

As a bremsstrahlung field, as a rule, multilayer electron shells of atoms.

7. The fine structure constant and electron precession

An electron in atomic physics is first of all a material particle and only then a carrier of electric charge. With a high degree of probability, one can expect the precession of the electron rotation axis relative to the motion trajectory τ. The precession frequency is usually denoted by the symbol Ω.



From the previous chapter, the energy recombination equation reduces to the speeds recombination equation:

$$x_{i+1}^3 + x_{i+1}^2(5 - 2x_i) + x_{i+1}(8 - 6x_i + x_i^2) - (4x_i - x_i^2) = 0,$$

Here: $x_i = \frac{v_i^2}{c^2}$; $x_{i+1} = \frac{v_{i+1}^2}{c^2}$; – ratios of squared speeds.

Let's designate neighboring speeds through v_1 and v_2 . The equation is converted to a simple form.

$$(x_2 - x_1 + 2)^2(1 + x_2) - 4 = 0.$$

We believe that the balance of variables and constants differs by a very small amount $|\pm\delta^2| \ll 1$.

Then:

$$(x_2 - x_1 + 2)^2(1 + x_2) - 4 = \pm\delta^2;$$

or $$(x_2 - x_1 + 2)^2 = \frac{4 \pm \delta^2}{1 + x_2}.$$

The rate recombination process should stop at positive values of the kinetic energy. We accept δ^2 with a negative sign. If the energy recombination equations claim the fundamental form of notation, then the following requirements must be satisfied for δ:

1. $\delta^2 -$ much less than unity;

2. $\delta -$ is chosen from among the dimensionless physical constants.

Only the fine-structure constant $\boldsymbol{\alpha}$, ($\alpha = 7.297\ 352\ 569\ 3 * 10^{-3}$) can claim the role of such a quantity.

Recombination of kinetic energy:

$$\frac{mv_1^2}{2} = \frac{mv_2^2}{2} + mc^2 - \frac{mc^2\sqrt{1-\frac{\alpha^2}{4}}}{\sqrt{1+\frac{v_2^2}{c^2}}} .$$

From the Taylor series expansion of the function $\sqrt{1-\alpha^2/4} \approx 1 - \alpha^2/8$.

Total kinetic energy of a relativistic particle:

$$E_k \cong \frac{mv^2}{2} + mc^2 - \frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}} - \frac{\alpha^2}{8}\frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}} ;$$

$$E_k = E_s + E_r - E_h .$$

Here:

$E_s -$ linear motion energy;

$E_r -$ point self-torsion energy;

$E_h -$ self-rotation precession energy.

The electron precession energy at, $v \approx c$, $\alpha^2 = 0{,}532514 \cdot 10^{-4}$:

$$E_h = \frac{9.11 \cdot 10^{-31}\ (29{,}979)^2 \cdot 10^{14}\ 0.532514 \cdot 10^{-4}}{8\sqrt{2} * 1.6022 \cdot 10^{-19}} = 2.41\ eV.$$

The electron precession energy at, $v \leq\ 10^7\ m/c$

$$E_h = \frac{9{,}11 \cdot 10^{-31}\ (29{,}979)^2 \cdot 10^{14}\ 0{,}532514 \cdot 10^{-4}}{8 * 1{,}6022 \cdot 10^{-19}} = 3.40\ eV.$$

Generalized recombination equation for velocity values. Each previous solution is an initial condition for the next solution:

$$x_{i+1}^3 + x_{i+1}^2(5 - 2x_i) + x_{i+1}\left(8 - 6x_i + x_i^2\right) - \left(4x_i - x_i^2 - \alpha^2\right) = 0.$$

The minimum relativistic velocity of a particle:

$$\left(4x_i - x_i^2 - \alpha^2\right) = 0;$$

$v_{i\,min} = 0{,}10946 \cdot 10^7\ \ m/s$ (Speed in the second orbit of the Bohr atom).

Calculation of threshold electron velocities, starting from the speed of light. The results depend on the initial braking speed:

№ quantum	1	2	3	4	5	6	7	8
$v_i * 10^7\ m/s$	30.000	22.993	17.101	12.449	8.942	6.375	4.526	3.206
№ quantum	9	10	11	12	13	14	15	16
$v_i * 10^7\ m/s$	2.268	1.603	1.131	0.796	0.558	0.387	0.262	0.169

Generalized mechanical momentum:

$$p \cong mv + \frac{mv}{\sqrt{1+\frac{v^2}{c^2}}}\left(1+\frac{\alpha^2}{8}\right).$$

Planck's hypothesis and bremsstrahlung energy:

$$\hbar\omega_r = mc^2 - \frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}}.$$

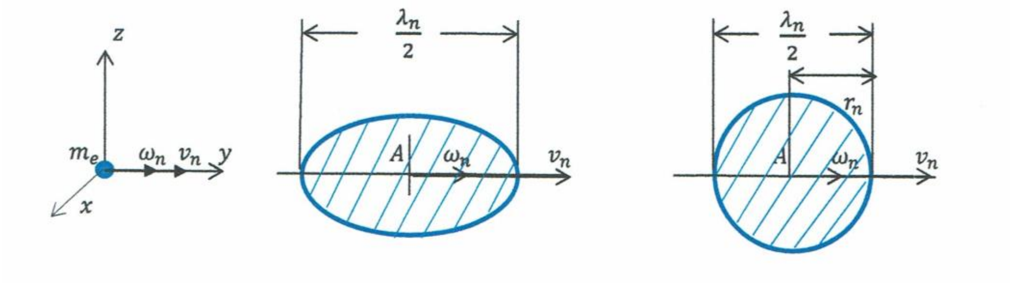
Planck's hypothesis and electron precession energy:

$$\hbar\omega_h = \frac{\alpha^2}{8}\frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}}.$$

As an assumption, the precession energy is the Cherenkov radiation.

8. The size of a free electron

Possible dimensions of an electron in a free (unbound) state. The electron moves in a constant electric field.



a) b) c)

a) **A material point** - the mass and charge of an electron are concentrated in an infinitely small volume of three-dimensional space.

b) **Ellipse** is an electromechanical plane standing wave. The mass and charge of the electron are distributed over the area of the ellipse.

c*)* **The disk** is a standing flat electromechanical wave of regular circular shape. The effective radius of the outer ring is $r_n \leq \lambda_n/2$. Translational speed of the center of the circle v_n. Angular speed of rotation of the disk relative to the transverse axis ω_n. The mass and charge are evenly distributed over the disk. Rotation about the transverse axis creates the effect of a spherical cloud.

The source of bremsstrahlung is the mechanical energy of the electron's own rotation.

Equality must be met.

$$I_n \frac{\omega_n^2}{2} = \hbar\omega_e = m_e c^2 - \frac{m_e c^2}{\sqrt{1+\frac{v_n^2}{c^2}}}.$$

n – energy recombination number.

The electron is affected by the recoil momentum of the outgoing X-ray photon. The particle is decelerated, and a single recombination of the total kinetic energy of the electron occurs. The angular mechanical velocity of the particle before recombination is equal to twice the cyclic frequency of the emitted photon:

$$\omega_n = 2\omega_e.$$

The intrinsic angular momentum of the electron:

$$L = I_n \omega_n = \hbar\,;$$

$$I_n \omega_e = \frac{1}{2}\hbar\,;$$

Here, $I_n -$ moment of inertia of an electron.

The intrinsic angular momentum of an electron is a constant value. For subsequent transformations, we use the known expressions:

$I_n = m_e r_n^2/4$, (thin round hard disk);

$$\omega_e = 2\pi\nu_e\,;$$

$$c = \nu_e \lambda_e\,.$$

Effective of electron radius:

$$r_n = \frac{1}{\pi}\sqrt{\frac{\Lambda * \lambda_e}{2}}.$$

Here: $\Lambda = \frac{h}{m_e \mathrm{c}} = 2{,}42631 \cdot 10^{-3}\ nm$, Compton wavelength of an electron.

Minimum X-ray photon wavelength ($\boldsymbol{v_{max} \cong c}$):

$$\hbar\omega_e = m_e c^2 - \frac{m_e c^2}{\sqrt{1+\frac{v_n^2}{c^2}}}\,;$$

$$\lambda_e = \Lambda\left(1 - \frac{1}{\sqrt{1+\frac{v^2}{c^2}}}\right)^{-1}\,;$$

$$\lambda_{e\,min} = 2{,}42631 \cdot 10^{-3}\left(1 - 1/\sqrt{2}\right)^{-1} = 8{,}284 \cdot 10^{-3}\ nm.$$

The minimum radius of an free electron:

$$r_{n\,min} = \frac{1}{\pi}\sqrt{\frac{\Lambda * \lambda_e}{2}} = \frac{1}{\pi}\sqrt{\frac{2.42631 * 8.284}{2}} \cdot 10^{-3} = 1{,}009 \cdot\ 10^{-3}\ nm = 0{,}0101\text{Å}$$

Maximum X-ray photon wavelength ($v_{min} = 0.10946 \cdot 10^7\ m/s$):

$$\hbar\omega_e = m_e c^2 - \frac{m_e c^2}{\sqrt{1 + \frac{v^2}{c^2}}} \cong \frac{m_e v^2}{2}\ ;$$

$$\lambda_e = \frac{2hc}{m_e v^2} = 2\Lambda\frac{c^2}{v^2}\,;$$

$$\lambda_{e\,max} = 2 \cdot 2.42631\frac{(29.9792458)^2}{(0.10946)^2} * 10^{-3} = 364\,005 \cdot 10^{-3}\ nm.$$

The maximum radius of an free electron:

$$r_{n\,max} = \frac{1}{\pi}\sqrt{\frac{\Lambda * \lambda_e}{2}} = \frac{1}{\pi}\sqrt{\frac{2.42631 * 364005}{2}} * 10^{-3} = 211{,}5 \cdot 10^{-3}\ nm \approx 2{,}12\ \text{Å}$$

Calculation of the electron radius starting from the speed of light.

$$x_{i+1}^3 + x_{i+1}^2(5 - 2x_i) + x_{i+1}\left(8 - 6x_i + x_i^2\right) - \left(4x_i - x_i^2 - \alpha^2\right) = 0.$$

Скорость электрона Electron speed $v_n\ * 10^7\ ms^{-1}$	Длина волн фотона Photon wavelength $\lambda_e\ * 10^{-3}\ nm$	Эффективный радиус электрона Effective electron radius $r_n\ * 10^{-3}\ nm$ $\leftrightarrow$	r_n Å
v_1 = 30.00	8.284	1.009	0.010
v_2 = 23.00	11.76	1.194	0.012
v_3 = 17.1	18.49	1.508	0.015
v_4 = 12.45	31.91	1.980	0.020
v_5 = 8.94	58.29	2.677	0.027
v_6 = 6.38	111.10	3.695	0.037
v_7 = 4.53	217.40	5.169	0.052
v_8 = 3.21	428.00	7.253	0.073
v_9 = 2.27	851.10	10.228	0.102
÷	÷	÷	÷

Calculation of electron radii in reverse order ($v_{min} = 0.10946 \cdot 10^7\ m/s$).

$$x_{i-1} = x_i + 2 - \sqrt{\frac{4 - \alpha^2}{1 + x_i}}\,.$$

The wavelength of an X-ray photon carries information about the size of an electron.

Electron speed $v_n \ * 10^7\ ms^{-1}$	Photon wavelength $\lambda_e \ \cdot 10^{-3}\ nm$	Effective electron radius $r_n \cdot 10^{-3}\ nm$ ↔	Å
$v_{16} = 0.10946$	364 005	211.50	2.12
$v_{15} = 0.18946$	121 502	122.21	1.22
$v_{14} = 0.28940$	52 074	80.01	0.80
$v_{13} = 0.42363$	24 302	54.65	0.55
$v_{12} = 0.60898$	11 760	38.02	0.38
$v_{11} = 0.86808$	5 788	26.67	0.27
$v_{10} = 1.23232$	2 872	18.79	0.19
$v_9 = 1.74564$	1 431	13.26	0.13
$v_8 = 2.46956$	715	9.37	0.09
÷	÷	÷	÷

An electron is a standing electromechanical wave that does not have stable dimensions. The maximum radius is comparable with the sizes of atoms.

9. On the structure of a relativistic electron

How is an electron arranged? The simple question with no direct answer. According to the VERSION of the Elementary Theory of Relativity, **the total kinetic energy** of an electron is determined by the expression:

$$E_k = E_s + E_r - E_h.$$

Here:

E_s – linear motion energy;

E_r – point self-torsion energy;

E_h – self-rotation precession energy.

For a **relativistic free** electron, the precession energy is very small and does not significantly affect its structure:

$$E_k = \frac{m_e v^2}{2} + m_e c^2 - \frac{m_e c^2}{\sqrt{1+\frac{v^2}{c^2}}}.$$

The ratio of the effective speed to the speed of light is equal to the tangent of some angle alpha, $\tan\alpha = v/c$. The negative value of the angle has no physical meaning.

$$E_k \cong \frac{1}{2} m_e c^2 \tan^2\alpha + 2m_e c^2 \sin^2\frac{\alpha}{2}.$$

Here, α – generalized angular parameter.

From the restrictions on the speed of light, the restrictions on the angle alpha follow:

$$0 \le \tan^2\alpha \le 1 \implies 0 \le \alpha \le \frac{\pi}{4}.$$

For small values of the angle α≪π⁄4, the approximate equality is fulfilled:

$$\alpha^2 \cong \frac{4}{3}\frac{E_k}{m_e c^2}\,.$$

All geometric parameters of the particle are hidden in the spatial angular coordinate α. The point energy of self-torsion is equal to:

$$E_r = 2m_e c^2 \sin^2\frac{\alpha}{2} = h\frac{c}{\lambda_e}\,.$$

Here:

m_e − invariant electron mass $\langle kg\rangle$;

h − Planck's constant $\langle Js\rangle$;

λ_e − possible radiation wavelength $\langle nm\rangle$.

Geometric interpretation.

X-ray photon wavelength:

$$\lambda_e = \frac{1}{2}\frac{\Lambda}{\sin^2\frac{\alpha}{2}}\,;$$

$$\alpha = 2\sin^{-1}\sqrt{\frac{\Lambda}{2\lambda_e}}\,.$$

Angle values for maximum and minimum X-ray wavelength; $\lambda_{min} = 8.284 \cdot 10^{-3} nm$; $\lambda_{max} = 364005 \cdot 10^{-3} nm$:

1. $\alpha_{max} = 2\sin^{-1}\sqrt{\frac{2.42631}{2*8.284}} = 44.999829^0 \approx 45^0 = \frac{2\pi}{8}\,;$

2. $\alpha_{min} = 2\sin^{-1}\sqrt{\frac{2.42631}{2*364005}} = 0.2091979^0 \approx 0.21^0 = \frac{2\pi}{1721}.$

Effective of electron radius:

$$r_n = \frac{1}{\pi}\sqrt{\frac{\Lambda*\lambda_e}{2}} = \frac{1}{\pi}\sqrt{\frac{\Lambda*\Lambda}{4\sin^2\frac{\alpha}{2}}} = \frac{1}{2\pi}\frac{\Lambda}{\sin\frac{\alpha}{2}}\,.$$

Perimeter of an electron circle:

$$L_n = 2\pi r_n = \frac{\Lambda}{\sin\frac{\alpha}{2}}\,.$$

Physically, exactly - **an electron is a body of rotation**. The Compton wavelength is equal to the perimeter of some coaxial circle inside the electron disk. The transverse rotation forms the Compton sphere. Sphere Radius:

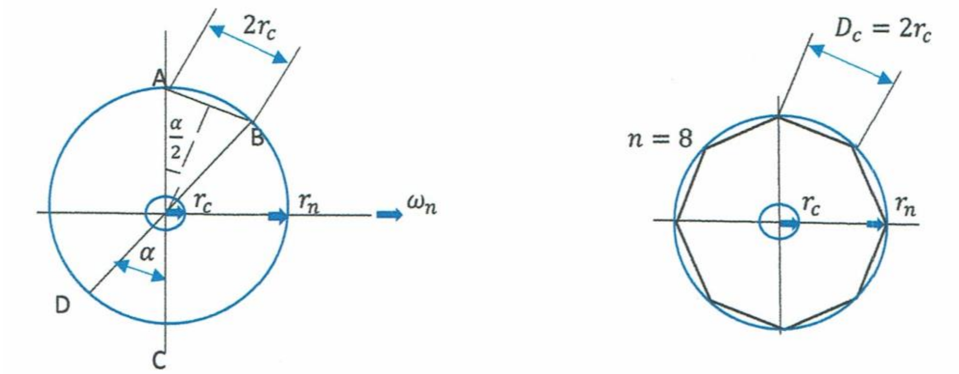
$$r_c = \frac{\Lambda}{2\pi} = \frac{2{,}42631*10^{-3} nm}{2\pi} = 0.38616 \cdot 10^{-3} nm.$$

The ratio of the radii of an electron and the Compton sphere has a simple form:

$$r_n \sin\frac{\alpha}{2} = r_c = const.$$

The formula for the relativistic energy of rotation is transformed into the following form:

$$E_r = 2m_e c^2 \sin^2\frac{\alpha}{2} = 2m_e c^2 \frac{r_c^2}{r_n^2}.$$



The radius of the circumscribed circle for any regular polygon is:

$$r_n = \frac{D_c}{2\sin\frac{\pi}{n}} = \frac{2r_c}{2\sin\frac{\pi}{n}} = \frac{r_c}{\sin\frac{\pi}{n}}\,. \qquad (10)$$

Here:

n – Number of sides of a regular polygon;

D_c – Side of a polygon (diameter of a Compton sphere).

For n≫8, the radius of an electron is determined by a simple formula:

$$r_n \cong \frac{r_c n}{\pi}; \qquad (8 \ll n \ll 1721).$$

Average values of the energy of a standing spherical wave and the relativistic energy of an electron:

$$E_r \sim \frac{1}{2} m A_0^2 \omega^2 \frac{r_0^2}{r^2};$$

$$E_r = 2m_e c^2 \frac{r_c^2}{r_n^2}\,.$$

From the comparison of the formulas, one can draw an unambiguous conclusion - inside the **relativistic** electron, between the Compton sphere and the outer sphere, a standing spherical wave pulsates. Regarding the amplitude, frequency and phase of oscillations, it is impossible to say anything definite.

10. Compton effect

According to the Bohr model, it is customary to assume that electrons rotate uniformly around a positively charged nucleus in stationary orbits of an atom. In the highest orbit, the "**energy of valence electrons**" is minimal. The generalized mechanical momentum of an electron is equal to zero $(\mathrm{p_e} = 0)$. The electron exists in the form of a standing spherical wave. Such electrons can be considered free. The collision of a free electron with a hard X-ray photon is called the Compton effect. As a result of the collision, mobile recoil electrons and X-ray photons of lower frequency appear. The transition of a free electron from a state of rest to a state of motion occurs in two stages.

1) At the first stage, the momentum and energy of the electron's own rotation are formed without linear motion:

$$p'_{er} = \frac{m_e v_s}{\sqrt{1+\frac{v_s^2}{c^2}}};$$

$$E'_{er} = m_e c^2 - \frac{m_e c^2}{\sqrt{1+\frac{v_s^2}{c^2}}}.$$

Here, v_s – starting fictitious speed.

2) At the second stage, momentum and energy are separated into classical and relativistic components (recombination):

$$p_e = p_{es} + p_{er} = m_e v + \frac{m_e v}{\sqrt{1+\frac{v^2}{c^2}}};$$

$$E_k = E_{es} + E_{er} = \frac{m_e v^2}{2} + m_e c^2 - \frac{m_e v^2}{\sqrt{1+\frac{v^2}{c^2}}}.$$

Here, v – effective linear speed of an electron.

From the equality of the energies of movement and start $(\mathrm{E_k} = \mathrm{E'_{er}})$ it is established to link between the starting speed and the effective speed of translational movement. We introduce additional dimensionless functions:

$$X(v) = \frac{1}{\sqrt{1+\frac{v^2}{c^2}}} \quad \text{и} \quad Y(v_s) = \frac{1}{\sqrt{1+\frac{v_s^2}{c^2}}}.$$

The functions are connected by two mutually inverse expressions:

$$Y = \frac{1}{2}\left(1 - \frac{1}{X^2}\right) + X;$$

$$X^3 + X^2(0.5 - Y) - 0.5 = 0.$$

Knowing one of the speeds, you can always calculate the second.

Collision of Particle.

Momentum and energy of a photon before and after collision with an electron:

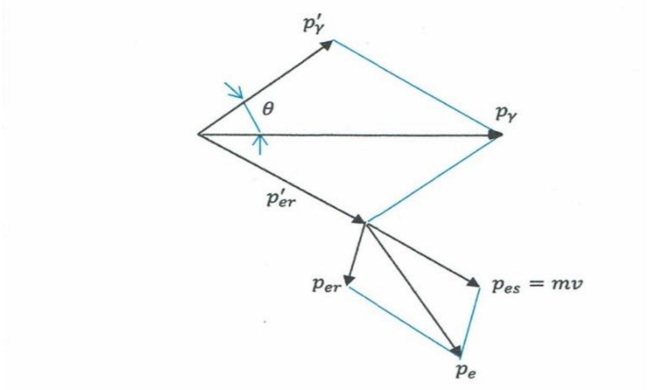
$$p_\gamma = \frac{h\nu}{c}\ ; \quad E_\gamma = h\nu\ ; \quad p'_\gamma = \frac{h\nu'}{c};\quad E'_\gamma = h\nu'\ .$$

Here:

h – Planck's constant $\langle J * s \rangle$;

ν – oscillation frequency of the incoming photon $\langle Hz \rangle$;

ν' – oscillation frequency of the scattered photon $\langle Hz \rangle$.



The balance of momentum and energy of the electron after the collision:

$$\vec{p}'_{er} = \vec{p}_\gamma - \vec{p}'_\gamma\ ;$$

$$E'_{er} = h\nu - h\nu'.$$

The relativistic components of momentum and energy are related by the relation (VERSION):

$$E'^2_{er} = 2m_e c^2 E'_{er} - p'^2_{er} c^2\ .$$

Here: m_e – the mass of an electron $\langle kg \rangle$;

c – the speed of light $\langle m * s^{-1} \rangle$.

Taking into account the relativistic relation and the collision diagram, a system of equations is formed:

$$2m_e c^2 E'_{er} - E'^2_{er} = h^2\nu^2 + h^2{\nu'}^2 - 2h^2\nu\nu'\cos\theta\ ;$$

$$E'_{er} = h\nu - h\nu'.$$

From the system of equations, the main expression for the Compton effect follows:

$$2m_e c^2(h\nu - h\nu') - (h\nu - h\nu')^2 = h^2\nu^2 + h^2{\nu'}^2 - 2h^2\nu\nu'\cos\theta.$$

Intermediate form of the equation:

$$\frac{m_e c^2}{h}\left(\frac{1}{\nu'}-\frac{1}{\nu}\right)=\frac{\nu^2+\nu'^2}{\nu\nu'}-(1+\cos\theta).$$

For close frequencies:

$$\lim_{\nu'\to\nu}\frac{\nu^2+\nu'^2}{\nu\nu'}\cong 2\,.$$

In simplified form.

$$\frac{m_e c^2}{h}\left(\frac{1}{\nu'}-\frac{1}{\nu}\right)=1-\cos\theta.$$

Wavelengths of incoming and scattered photons:

$$\lambda=\frac{c}{\nu};\qquad \lambda'=\frac{c}{\nu'}\,.$$

Wavelength difference:

$$\Delta\lambda=\lambda'-\lambda=\frac{h}{m_e c}(1-\cos\theta)=2\Lambda\, sin^2\frac{\theta}{2}\,.$$

Here $\Lambda=\frac{h}{m_e c}=2.426\cdot 10^{-3}\,nm$ – **The Compton wavelength** for an electron in nanometers.

The difference $\Delta\lambda=\lambda'-\lambda$ does not depend on the initial wavelength λ and the nature of the scattering substance. The formula is valid in the range of hard X-ray waves$10\cdot\ 10^{-3}\mathrm{nm}\le\lambda\le 10\ \mathrm{nm}$. For comparison: the radius of the first orbit of the hydrogen atom is $\mathrm{r}_1=53\cdot 10^{-3}\mathrm{nm}$. The sizes of atoms of various elements in a solid body are about $1\text{Å}=0.1\ \mathrm{nm}=100\cdot 10^{-3}\mathrm{nm}$. Thus, the simplification option is close to reality, the Compton effect is confirmed.

The energy balance in the Compton effect with scattered photons and recoil electrons can be written in the following form:

$$\frac{hc}{\lambda}-\frac{hc}{\lambda'}=m_e c^2-\frac{m_e c^2}{\sqrt{1+\frac{v_s^2}{c^2}}}\ .$$

Starting speed.

We write the last equation in the following form:

$$\frac{h}{m_e c}\frac{\Delta\lambda}{\lambda_c^2}=1-\frac{1}{\sqrt{1+\frac{v_s^2}{c^2}}};$$

Here $\boldsymbol{\lambda_c}=\sqrt{\boldsymbol{\lambda}\cdot\boldsymbol{\lambda'}}$ – mean geometric wavelength of incoming and scattered photons.

We use the Compton equation and get a useful expression:

$$2\left(\frac{\Lambda\sin\frac{\theta}{2}}{\lambda_c}\right)^2=1-\frac{1}{\sqrt{1+\frac{v_s^2}{c^2}}}=1-Y(v_s)\,.$$

From this expression, one can determine the **starting speed** of an electron after its interaction with an incoming photon (in the X-ray range of waves). **For each photon wavelength and reflection angle, there will be a recoil electron of its own in terms of velocity**.

An example of calculating the speed of a recoil electron.

$\lambda = 70 \cdot 10^{-3} nm$	$\lambda' = \lambda + 2\Lambda\, sin^2 \frac{\theta}{2}$, The wavelength of the scattered photon.
$\theta = 110^0$	$\lambda' = 70 \cdot\ 10^{-3} + 2 \cdot 2{,}4263 \cdot\ 10^{-3} sin^2 \frac{110^0}{2} = 73{,}256\ \cdot 10^{-3} nm.$
$v_s = ?, v = ?.$	

Mean geometric wavelength of photons.

$$\lambda_c = \sqrt{\lambda\, \lambda'} = \sqrt{70 * 73{,}256} \cdot 10^{-3} = 71{,}609 \cdot\ 10^{-3} nm.$$

According to the formula:

$$2\left(\frac{\Lambda \sin\frac{\theta}{2}}{\lambda_c}\right)^2 = 2\left(\frac{2{,}4263 \cdot 10^{-3}}{71{,}609 \cdot 10^{-3}} \sin\frac{110^0}{2}\right)^2 = 0{,}001541.$$

$$\sqrt{1 + \frac{v_s^2}{c^2}} = \frac{1}{1 - 0{,}001541} = 1{,}0015434; \quad Y(v_s) = 1 - 0{,}001541 = 0{,}998459;$$

$$\frac{v_s^2}{c^2} = (1{,}0015434)^2 - 1 = 0{,}03089\, ; \quad \frac{v_s}{c} = \sqrt{0{,}03089} = 0{,}055580\, .$$

We solve the cubic equation:

$$X^3 + X^2(0.5 - 0.998459) - 0.5 = 0; \qquad X(v) = 0{,}99923;$$

$$\frac{v^2}{c^2} = \frac{1}{X^2} - 1 = \frac{1}{0.99923^2} - 1 = 0.0015418; \quad \frac{v}{c} = \sqrt{0.0015418} = 0{,}039266.$$

Answer:

$v_s = 30 \cdot 10^7\, m/s * 0{,}05558 = 1{,}6674 \cdot 10^7\, m/s$; starting speed;

$v = 30 \cdot 10^7\, m/s * 0.039266 = 1{,}1780 \cdot 10^7\, m/s$; effective speed.

A recoil electron knocked out of orbit is relativistic.

From the Heisenberg uncertainty relation:

$$\Delta E_r \cdot \Delta t \geq h.$$

$$\Delta E_r \approx E'_{er} - E_{er} = E_s = \frac{m_e v^2}{2}.$$

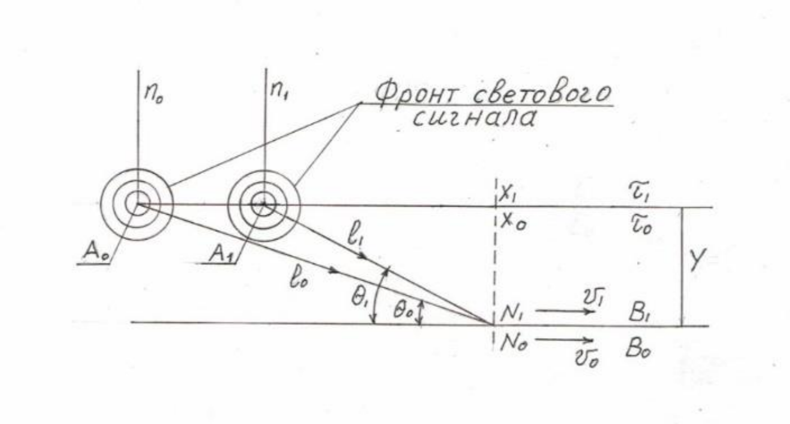
Start time (relaxation):

$$\Delta t \approx \frac{2h}{m_e v^2};$$

$$\Delta t \approx \frac{2 * 6{,}626 \cdot 10^{-34}}{9{,}11 \cdot 10^{-31} * 1{,}178^2 \cdot 10^{14}} = 1{,}05 \cdot 10^{-17} s.$$

11. Aberration of Light

The apparent displacement of a stationary object when observed in different frames of reference is called the aberration of light. The figure shows a graphical illustration of such an offset. Depending on the speed of the light receiver, the angle of observation of the object above the horizon changes. The origins of the base and parallel reference systems are taken as fixed points. Fixed points radiate a light signal, which is captured by the movable observers $N_0(N_1)$ of the corresponding reference systems. Observers coincide in absolute space and time of a single reference system. For each value v_0 in the base frame of reference there is its own parallel frame of reference. Light differs into one's own and another's.



Trajectories B_1, B_0 are spaced from the axes τ_1, τ_0 at the same distance Y of absolute space, $l_1, l_0 -$ are the distances between the centers of coordinates and observers in their own reference frames, $\theta_1, \theta_0 -$ observation angles in own reference systems.

Cosines of observation angles:

$$\cos\theta_0 = \frac{x_0}{l_0} = \frac{x_0}{c_0 t_0}; \quad \cos\theta_1 = \frac{x_1}{l_1} = \frac{x_1}{c_1 t_1}.$$

Here:

$c_0 = c -$ the speed of light in the base frame of reference ($\approx 30 * 10^7\ m/s$);

$c_1 = c/\left(1+\frac{v_0^2}{c^2}\right) -$ the speed of light in a parallel frame of reference.

For observer N_0, direct transformations of Lorentz coordinates are used in the VERSION variant:

$$\cos\theta_0 = \frac{x_0}{c_0 t_0} = \frac{\frac{x_1}{ct_1}\left(1+\frac{v_0^2}{c^2}\right)^{\frac{1}{2}} + \frac{v_0}{c}\left(1+\frac{v_0^2}{c^2}\right)^{-\frac{1}{2}}}{\left(1+\frac{v_0^2}{c^2}\right)^{-\frac{1}{2}} + \frac{x_1 v_0}{ct_1 c}\left(1+\frac{v_0^2}{c^2}\right)^{\frac{1}{2}}}; \quad \left(\frac{v_0}{c} = \beta\right);$$

$$\cos\theta_0 = \frac{\beta + \cos\theta_1}{1+\beta\cos\theta_1}; \qquad \sin\theta_0 = \sqrt{1-cos^2\theta_0}.$$

For observer N_1, inverse transformations of Lorentz coordinates are used in the VERSION variant:

$$\cos\theta_1 = \frac{x_1}{c_1 t_1} = \frac{x_0 - v_0 t_0}{t_0 - x_0\frac{v_0}{c^2}}\,\frac{1}{\left(1+\frac{v_0^2}{c^2}\right)}\,\frac{1}{c_1} = \frac{\frac{x_0}{ct_0}-\frac{v_0}{c}}{1-\frac{x_0}{ct_0}\frac{v_0}{c}}; \quad \left(\frac{v_0}{c} = \beta\right);$$

$$\cos\theta_1 = \frac{\cos\theta_0 - \beta}{1-\beta\cos\theta_0}; \qquad \sin\theta_1 = \sqrt{1-cos^2\theta_1}\ .$$

The formulas for the observation angles in the Elementary Theory of Relativity and in the Special Theory of Relativity coincide completely:

$$\cos\theta_0 = \frac{\beta+\cos\theta_1}{1+\beta\cos\theta_1}; \qquad \cos\theta_1 = \frac{\cos\theta_0-\beta}{1-\beta\cos\theta_0};$$

$$\sin\theta_0 = \frac{\sqrt{1-\beta^2}\ \sin\theta_1}{1+\beta\cos\theta_1}; \qquad \sin\theta_1 = \frac{\sqrt{1-\beta^2}\ \sin\theta_0}{1-\beta\cos\theta_0}.$$

Neglecting all powers of β^n except for the first one (β≪1), we obtain an approximate expression for the connection of observation angles. The cosines of small angles can be taken as unity:

$$\sin\theta_1 \cong \sin\theta_0\frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta_0} \cong \sin\theta_0\frac{\sqrt{1-\beta^2}}{1-\beta} \cong \sin\theta_0\frac{1}{1-\beta};$$

$$\sin\theta_1 \cong \sin\theta_0\frac{1+\beta}{1-\beta^2} \cong (1+\beta)\sin\theta_0.$$

For small angles, the following equalities hold: $\sin\theta_1 \cong \theta_1$ и$/and$ $\sin\theta_0 \cong \theta_0$.

$$\sin\theta_1 - \sin\theta_0 \cong \theta_1 - \theta_0 = \alpha.$$

The difference in viewing angles is called light aberration. $\boldsymbol{\alpha = \theta_1 - \theta_0}$.

$$\alpha \cong \beta\sin\theta_0 = \frac{v_0}{\mathrm{c}}\sin\theta_0 = k\sin\theta_0.$$

A more accurate value of aberration is given by the formula:

$$\alpha \cong \left(\sqrt{\frac{1+\beta}{1-\beta}} - 1\right)\sin\theta_0 = k\sin\theta_0.$$

The annual aberration constant **k** for the average orbital velocity of the Earth, adopted by the International Astronomical Union, is 20.49552" arc seconds.

12. Relativistic Addition for External Photoelectric Effect

Classification of speeds.

The speeds of translational motion of elementary particles are divided into three main ranges: classical; relativistic; ultra-relativistic.

1. Classical, $0 \le v \le 1.094\cdot 10^6\ ms^{-1}$.

2. Relativistic, $1.094\cdot 10^6 \le v \le 10\cdot 10^6\ ms^{-1}$.

3. Ultra-relativistic, $10 \cdot 10^6 \leq v \leq 300 \cdot 10^6\ ms^{-1}$.

The Elementary Theory of Relativity (ETR) establishes the exact value of the boundary between classical and relativistic velocities, $v = 1.0946 \cdot 10^6\ ms^{-1}$ [1]. The boundary between the relativistic and ultra-relativistic ranges has a conventional value, $v = 10 \cdot 10^6\ ms^{-1}$.

Classification of energies.

Form of recording the total kinetic energy of a particle in the Elementary Theory of Relativity [2]:

$$E_k = E_s + E_r - E_h. \qquad (1)$$

Here:

$E_s = \frac{mv^2}{2}$; $-$ energy of translational motion a particle;

$E_r = mc^2 - \frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}}$; $-$ energy of self-torsion of a particle;

$E_h = \frac{\alpha^2}{8}\frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}}$; $-$ precession energy of self-torsion of a particle.

The form of recording kinetic energy can be simplified depending on the location of the particle in a certain speed range. The transition of a particle from a relativistic state to a classical state occurs abruptly: $E_r - E_h = 0; v = 1.094 \cdot 10^6\ ms^{-1}$. For an electron in vacuum, two main forms of recording the total kinetic energy can be distinguished.

1) Classic formula:

$$E_k = \frac{m_e v^2}{2}.$$

2) Relativistic formula:

$$E_s = \frac{m_e v^2}{2} ;$$

$$E_r = m_e c^2 - \frac{m_e c^2}{\sqrt{1+\frac{v^2}{c^2}}} \cong \frac{m_e v^2}{2} ; \quad E_h = \frac{\alpha^2}{8}\frac{m_e c^2}{\sqrt{1+\frac{v^2}{c^2}}} \cong 3.4\ eV ;$$

$$E_k = E_s + E_r - E_h \cong m_e v^2 - 3.4\ eV. \qquad (2)$$

The precession energy is strongly suppressed by the fine structure constant **α** and weakly depends on the translational speed. The numerical value $E_h = 3{,}4\ eV$ is equal to the average value of this energy at the boundaries of the relativistic speeds range. When describing the external photoelectric effect, **ultra-relativistic energy and velocity intervals are not used**. All types of energy are expressed in non-systemic units - electron volts.

External photoelectric effect.

The external photoelectric effect is well described in the existing scientific and technical literature and does need additional comments. Einstein's equation describes the external photoelectric effect for the non-relativistic (classical) case of photoelectrons motion outside a conductor. Photoelectrons are knocked out of the metal by the action of quanta of external electromagnetic radiation. The range of photon energies that cause photocurrent in a vacuum corresponds to violet, ultraviolet and ultra-soft X-rays (less than 0.5 keV) [3]. The speeds of ejected electrons can significantly exceed the upper limit of the classical speed range. The question arises of how to use Einstein's equation in the case of relativistic motion of photoelectrons outside a conductor.

1) Consider the classical Einstein equation in non-relativistic form:

$$h\nu = E_k + \phi = \frac{m_e v_{max}^2}{2} + \phi \,. \tag{3}$$

Here: ϕ is the work function of electron leaving the conductor (eV); $h\nu$ is the energy of a photon incident on a conductor (eV); E_k is the classical kinetic energy of an electron in vacuum (eV); is the electron mass (kg); v_{max} is the **maximum classical** electron speed; is **limited from above** by the relativistic value $1.094 \cdot 10^6\ ms^{-1}$. From the equation and experiments, the electron work function and the wavelength of the red boundary of the photoelectric effect are determined, $\lambda_0 = hc/\phi$. The speed of a classical photoelectron in vacuum is determined by the formula:

$$v_{max} = \sqrt{\frac{2(h\nu - \phi)}{m_e} 1.6022 \cdot 10^{-19}};\ \ ms^{-1} \,. \tag{4}$$

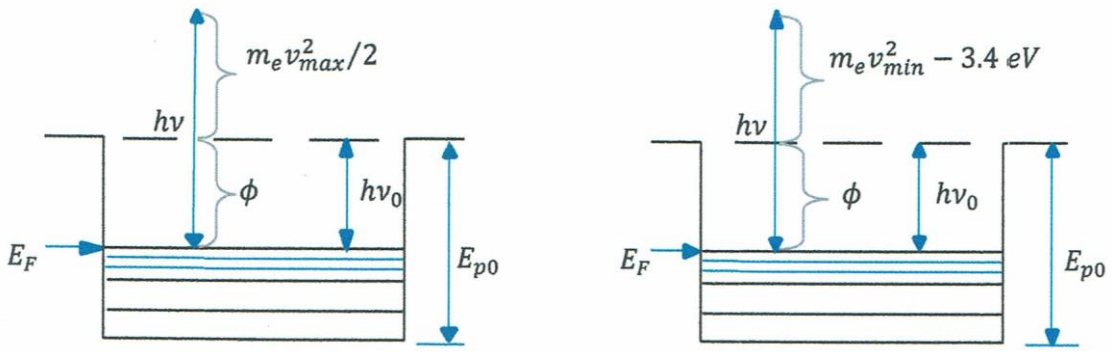
2) Let us replace, in Einstein's equation, the classical formula for recording kinetic energy with its relativistic version (2):

$$h\nu = E_k + \phi = m_e v_{min}^2 - 3.4\ eV + \phi. \tag{5}$$

Speed of a relativistic photoelectron in vacuum:

$$v_{min} = \sqrt{\frac{h\nu + 3.4 - \phi}{m_e} 1.6022 \cdot 10^{-19}}\ ;\ \ ms^{-1}. \tag{6}$$

Here: v_{min} is the **minimum relativistic** speed of the electron; **limited below** by the value $1.094 \cdot 10^6\ ms^{-1}$. Below is a graphical illustration of the photon and electron energy distributions taking into account the zone theory of metals. The temperature of the electron gas inside the metal is assumed to be equal to absolute zero, $T = 0\ K$.



A) Classic distribution. B) Relativistic distribution.

Here: E_F is the energy Fermi (level) for electrons; separates the valence band from the conduction band; Ep_0 is the depth of the potential well in the metal. For metals, the Fermi energy, at room temperature, is in the range of 3 to 10 eV. The transition of electrons from a classic state to a relativistic state can be demonstrated by calculating the speed of cesium (Cs) photoelectrons. The work function for cesium is 1.9 eV. The red limit is 653 nm. Below are the calculation results.

Photon energy, E_γ eV.	Wavelength, λ nm.	Frequency, $\nu * 10^{14}$ Hz.	Speed, v km/s.	
			Classic case, v_{max}	Relativistic case, v_{min}
1.9	653	4.59	**0**	773*
2.0	620	4.84	**188**	785*
2.5	496	6.05	**459**	834*
3.0	414	7.25	**622**	890*
3.5	354	8.46	**750**	938*
4.0	310	9.67	**859**	984*
4.5	276	10.88	**956**	1055*
5.0	248	12.09	**1090**	1069*
5.3	**234**	**12.82**	**1094**	**1094**
5.5	226	13.30	1125*	**1110**
6.0	207	14.51	1201*	**1149**
6.5	191	15.72	1272*	**1186**
7.0	177	16.93	1339*	**1223**
7.5	165	18.13	1403*	**1258**
8.0	155	19.34	1465*	**1293**
÷	÷	÷	÷	÷
÷	÷	÷	÷	÷
567.1	**2.19**	**1370**	14 100*	**10 000**

In the table, in bold type, real Classical and relativistic values speeds are marked. Asterisks indicate fictitious values of the corresponding speeds.

With a characteristic photon energy of 5.3 eV, the classical velocities of cesium photoelectrons acquire a relativistic character. The speed of electron transition to a relativistic state is always constant, v≅1094 km/s. The characteristic energies of photons depend on the nature of the irradiated conductor and the cleanliness of its surface. The values of characteristic energies are determined by a simple formula, $h\nu = 3.4\ eV + \phi$. The process of transition of a particle from a relativistic state to a classical state and back is very similar to the latent thermal energy of aggregate transformations of matter. For an electron at $v_{max} = v_{min}$, the latent transition energy is zero:

$$Q_e = E_r - E_h = 0; \quad (7)$$

$$Q_e = \frac{m_e v^2}{2*1.6022\cdot 10^{-19}} - 3{,}4\ eV = \frac{9.11\cdot 10^{-31}\cdot (1.094)^2\cdot 10^{12}}{2*1.6022\cdot 10^{-19}} - 3.4\ eV = 0.$$

13. Reference systems in relativistic mechanics

Physical reference system - a set of reference body, coordinate system and method of measuring time (hours). Conventionally, reference systems differ in:

— inertial and non-inertial systems;

— moving and non-moving systems;

— equal and unequal systems.

The concept of reference systems is a scientific abstraction. Without abstract thinking and mathematical models, a person, in principle, cannot study nature. The elementary theory of relativity considers a version of fixed, inertial frames of reference. Separation of the coordinate system from the reference body is not allowed. The concept of parallel reference systems is used, one of which is taken as the base one. Each frame of reference is represented by its own standards: lengths; time; masses; a reference body in the center of coordinates and a moving material point. There is an agreement: - **alternative material points, having different measurement scales, always coincide in the absolute space of a single reference system**. A particular case of shifting coordinate systems is described by Lorentz transformations**. Direct and inverse coordinate transformations are connected by mathematical formalism and do not reflect the physical difference between the events taking place.**

Relative motion is called motion in parallel reference systems. The first principle of relativity:

1. In dimensionless (relative) units, any physical processes proceed in the same way in all inertial frames of reference.

The theory is based on the **generalized** relative speed of interaction of connected alternative points - △. The relative speed is **directed across the movement** and is equal in absolute value to:

$$|\vec{\triangle}| = \frac{v_1 t_1}{t_0} = \frac{v_0 t_0}{t_1} = \sqrt{v_1 v_0}$$

For practical purposes, one uses its **own relative velocity** in a parallel (adjacent) frame of reference, $\triangle_1 = \mathrm{v}_0/\sqrt{1+\mathrm{v}_0^2/\mathrm{c}^2}$. The difference between the generalized and own relative speed is revealed at the beginning of the article. In the dynamic mode, the principle of the exchange of impulses of relative forces along the trajectory is introduced. The second principle of relativity:

2. The force impulse from the neighboring frame of reference must be balanced by the corresponding change in the momentum of a material point in its own frame of reference.

With **zero** initial conditions and **inequality** of reference systems, the record of this principle (one-dimensional case) looks like this:

$$F_{\tau 0} t_0 = m_1 v_1 = m_1 \triangle_1 \;;$$

$$F_{\tau 1} t_1 = m_0 v_0 \;;$$

$$F_{\tau 1} = F_{\tau 0}$$ (Newton's Third Law of Motion).

The equality of the relative force modules does not depend on the metric of the reference systems. Newton's forces, in their own frames of reference, are not relative forces.

In Euclidean space, the speeds of alternative points along the trajectory of motion are related by relations:

$$v_1 = v_0 / \left(1 + \frac{v_0^2}{c^2}\right);$$

$$v_{1x} = v_{0x} / \left(1 + \frac{v_0^2}{c^2}\right);$$

$$v_{1y} = v_{0y} / \left(1 + \frac{v_0^2}{c^2}\right);$$

$$v_{1z} = v_{0z} / \left(1 + \frac{v_0^2}{c^2}\right).$$

Dimensional physical constants in a parallel reference system are used in a modified form. EXAMPLES:

The speed of light.

$$c_1 = c\left(1 + \frac{v_0^2}{c^2}\right)^{-1} \langle \frac{m_0}{s_0} \rangle.$$

The specific density of the material.

$$\rho_1 = \rho_0 \left(1 + \frac{v_0^2}{c^2}\right)^2 \frac{kg_0}{m_0^3}.$$

Planck's constant.

$$h_1 = h\left(1 + \frac{v_0^2}{c^2}\right)^{-1} J_0 s_0, \langle \frac{kg_0 m_0^2}{s_0} \rangle.$$

The mass of the reference body at the center of coordinates:

$$\frac{m_1}{M_1} = \frac{m_0}{M_0};$$

$$M_1 = M_0 \frac{m_1}{m_0} = M_0 \sqrt{1 + \frac{v_0^2}{c^2}} \; kg_0 \,.$$

Fine structure constant in atomic physics:

$$\boldsymbol{\alpha_1 = \alpha_0 = \alpha}.$$

Gravitational constant:

$$G_1 = G_0\left(1+\frac{v_0^2}{c^2}\right)^{-2} \frac{m_0^3}{kg_0 s_0^2}.$$

Absolute temperature, invariant in neighboring reference systems:

$$T_1 = T_0 = T; \quad K.$$

Boltzmann constant,

$$k_{B1} = k_{B0}\left(1+\frac{v_0^2}{c^2}\right)^{-\frac{3}{2}}, \quad \frac{J_0}{K}.$$

The pressure of an ideal relativistic gas,

$p_1 \equiv p_0$ (numerically, invariants in neighboring reference frames).

Concentrations of particles of an ideal relativistic gas:

$N = n_1 V_1 = n_0 V_0$ (invariant, number of particles in neighboring frames of reference);

$$n_1 = n_0\left(1+\frac{v_0^2}{c^2}\right)^{\frac{3}{2}} \quad m_0^{-3}.$$

Everything is logical. The volume decreases, the concentration increases. At this stage, the author limits himself to the results obtained.

Generalized momentum and kinetic energy.

In a closed mechanical system, a group of points can move with different linear speeds. It is possible to summarize the generalized impulses of all points only in the laboratory (basic) reference system. The law of conservation of the geometric sum of impulses in a closed mechanical system must be fulfilled, $\sum \vec{p}_i$ = const. On the complex plane, for a beam **of coherent** particles moving in the **same direction**:

$$\sum_i \vec{p}_i = \sum_i m_i v_i + j\sum_i \frac{m_i v_i}{\sqrt{1+\frac{v_i^2}{c^2}}};$$

$$\sum_i \vec{p}_i = \sum_i p_{si} + j\sum_i p_{ri}.$$

Coherence is understood as the same directions of rotation of particles relative to the vector of their own linear velocity. The total energy of motion of a group **of coherent** particles in a closed mechanical system remains unchanged:

$$\sum E_{ki} = \sum_i \frac{m_i v_i^2}{2} + \sum_i\left(m_i c^2 - \frac{m_i c^2}{\sqrt{1-\frac{v_i^2}{c^2}}}\right);$$

$$\sum_i E_{ki} = \sum_i E_{si} + \sum_i E_{ri}$$

The general case of motion and collision of particles with different coherence is not considered in this article. Inertial reference systems are convenient and useful mathematical models of the real motion of mass bodies in space and time. They make it possible to reveal the features of the motion of particles at speeds close to the speed of light. All three Newton's laws must be fulfilled in the models.

14. List of basic parameters of a relativistic particle

1. Effective particle mass. $M = m + \frac{m}{\sqrt{1+\frac{v^2}{c^2}}}\left(1+\frac{\alpha^2}{8}\right)$; α – The fine-structure constant.
2. Generalized mechanical impulse. $p \cong mv + \frac{mv}{\sqrt{1+\frac{v^2}{c^2}}}\left(1+\frac{\alpha^2}{8}\right)$.
3. Total kinetic energy of the particle. $E_k \cong \frac{mv^2}{2} + mc^2 - \frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}} - \frac{\alpha^2}{8}\frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}}$. $E_k = E_s + E_r - E_h$.
4. Kinetic energy of translational motion. $E_s = \frac{mv^2}{2}$.
5. Energy of self-torsion of a particle. $E_r = mc^2 - \frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}} = \hbar\omega_r$; $\hbar$ – Dirac constant.
6. Precession energy of self-torsion of a particle. $E_h = \frac{\alpha^2}{8}\frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}} = \hbar\omega_h$.
7. Range of relativistic speeds. $0.10946 \cdot 10^7 ms^{-1} \ll v < c \cong 30 \cdot 10^7\ ms^{-1}$

15. Sources of Information

The derivations of the formulas and additional information are presented on the website of the Elementary Theory of Relativity, (https://halmich.ru).

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