Relativistic Addition for External Photoelectric Effect

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ANNOTATION

The photoelectric effect is described by Einstein's equation based on quantum concepts. The velocities of photoelectrons knocked out of matter are assumed to be non-relativistic (classical). This article discusses the Einstein equation in the range of relativistic speeds, i.e. for values that can already be compared with the speed of light.

Content

1. Classification of speeds

The speeds of translational motion of elementary particles are divided into three main ranges: classical; relativistic; ultra-relativistic.

1. Classical, $0 \le v \le 1.094 \cdot 10^6\ ms^{-1}$.

2. Relativistic, $1.094 \cdot 10^6 \le v \le 10 \cdot 10^6\ ms^{-1}$.

3. Ultra-relativistic, $10 \cdot 10^6 \le v \le 300 \cdot 10^6\ ms^{-1}$.

The Elementary Theory of Relativity (ETR) establishes the exact value of the boundary between classical and relativistic velocities, $v = 1.0946 \cdot 10^6\ ms^{-1}$ [1]. The boundary between the relativistic and ultra-relativistic ranges has a conventional value, $v = 10 \cdot 10^6\ ms^{-1}$.

2. Classification of energies

Form of recording the total kinetic energy of a particle in the Elementary Theory of Relativity [2]:

$$E_k = E_s + E_r - E_h. \qquad (1)$$

Here:

$E_s = \frac{mv^2}{2}$; $-$ energy of translational motion a particle;

$E_r = mc^2 - \frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}}$; $-$ energy of self-torsion of a particle;

$E_h = \frac{\alpha^2}{8}\frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}}$; $-$ precession energy of self-torsion of a particle.

The form of recording kinetic energy can be simplified depending on the location of the particle in a certain speed range. The transition of a particle from a relativistic state to a classical state occurs abruptly: $E_r = 0;\ E_h = 0; v = 1.094 \cdot 10^6\ ms^{-1}$. For an electron in vacuum, two main forms of recording the total kinetic energy can be distinguished.

1) Classic formula:

$$E_k = \frac{m_e v^2}{2}.$$

2) Relativistic formula:

$$E_s = \frac{m_e v^2}{2};$$

$$E_r = m_e c^2 - \frac{m_e c^2}{\sqrt{1+\frac{v^2}{c^2}}} \cong \frac{m_e v^2}{2}; \quad E_h = \frac{\alpha^2}{8}\frac{m_e c^2}{\sqrt{1+\frac{v^2}{c^2}}} \cong 3.4\ eV;$$

$$E_k = E_s + E_r - E_h \cong m_e v^2 - 3.4\ eV. \qquad (2)$$

The precession energy is strongly suppressed by the fine structure constant **α** and weakly depends on the translational speed. The numerical value $E_h = 3{,}4\ eV$ is equal to the average value of this energy at the boundaries of the relativistic speeds range. When describing the external photoelectric effect, **ultra-relativistic energy and velocity intervals are not used**. All types of energy are expressed in non-systemic units - electron volts.

3. External photoelectric effect

The external photoelectric effect is well described in the existing scientific and technical literature and does need additional comments. Einstein's equation describes the external photoelectric effect for the non-relativistic (classical) case of photoelectrons motion outside a conductor. Photoelectrons are knocked out of the metal by the action of quanta of external electromagnetic radiation. The range of photon energies that cause photocurrent in a vacuum corresponds to violet, ultraviolet and ultra-soft X-rays (less than 0.5 keV) [3]. The speeds of ejected electrons can significantly exceed the upper limit of the classical speed range. The question arises of how to use Einstein's equation in the case of relativistic motion of photoelectrons outside a conductor.

1) Consider the classical Einstein equation in non-relativistic form:

$$h\nu = E_k + \phi = \frac{m_e v_{max}^2}{2} + \phi\ . \qquad (3)$$

Here: ϕ is the work function of electron leaving the conductor (eV); $h\nu$ is the energy of a photon incident on a conductor (eV); E_k is the classical kinetic energy of an electron in vacuum (eV); is the electron mass (kg); v_{max} is the **maximum classical** electron speed; is **limited from above** by the relativistic value $1.094 \cdot 10^6\ ms^{-1}$. From the equation and experiments, the electron work function and the wavelength of the red boundary of the photoelectric effect are determined, $\lambda_0 = hc/\phi$. The speed of a classical photoelectron in vacuum is determined by the formula:

$$v_{max} = \sqrt{\frac{2(h\nu - \phi)}{m_e} 1.6022 \cdot 10^{-19}};\ \ ms^{-1}\ . \qquad (4)$$

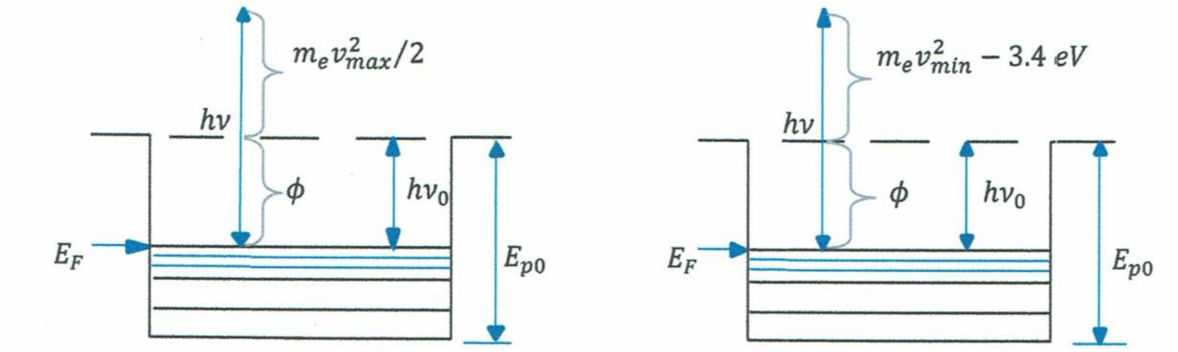
2) Let us replace, in Einstein's equation, the classical formula for recording kinetic energy with its relativistic version (2):

$$h\nu = E_k + \phi = m_e v_{min}^2 - 3.4\ eV + \phi. \qquad (5)$$

Speed of a relativistic photoelectron in vacuum:

$$v_{min} = \sqrt{\frac{h\nu + 3.4 - \phi}{m_e} 1.6022 \cdot 10^{-19}}\ ;\ \ ms^{-1}. \qquad (6)$$

Here: v_{min} is the **minimum relativistic** speed of the electron; **limited below** by the value $1.094 \cdot 10^6\ ms^{-1}$. Below is a graphical illustration of the photon and electron energy distributions taking into account the zone theory of metals. The temperature of the electron gas inside the metal is assumed to be equal to absolute zero, $T = 0\ K$.



A) Classic distribution. B) Relativistic distribution.

Here: E_F is the energy Fermi (level) for electrons; separates the valence band from the conduction band; Ep_0 is the depth of the potential well in the metal. For metals, the Fermi energy, at room temperature, is in the range of 3 to 10 eV. The transition of electrons from a classic state to a relativistic state can be demonstrated by calculating the speed of cesium (Cs) photoelectrons. The work function for cesium is 1.9 eV. The red limit is 653 nm. Below are the calculation results.

Photon energy, E_γ eV.	Wavelength, λ nm.	Frequency, $\nu * 10^{14}$ Hz.	Speed, v km/s.	
			Classic case, v_{max}	Relativistic case, v_{min}
1.9	653	4.59	**0**	773*
2.0	620	4.84	**188**	785*
2.5	496	6.05	**459**	834*
3.0	414	7.25	**622**	890*
3.5	354	8.46	**750**	938*
4.0	310	9.67	**859**	984*
4.5	276	10.88	**956**	1055*
5.0	248	12.09	**1090**	1069*
5.3	**234**	**12.82**	**1094**	**1094**
5.5	226	13.30	1125*	**1110**
6.0	207	14.51	1201*	**1149**
6.5	191	15.72	1272*	**1186**
7.0	177	16.93	1339*	**1223**
7.5	165	18.13	1403*	**1258**
8.0	155	19.34	1465*	**1293**
÷	÷	÷	÷	÷
÷	÷	÷	÷	÷
567.1	**2.19**	**1370**	14 100*	**10 000**

In the table, in bold type, real classical and relativistic values speeds are marked. Asterisks indicate fictitious values of the corresponding speeds.

With a characteristic photon energy of 5.3 eV, the classical velocities of cesium photoelectrons acquire a relativistic character. The speed of electron transition to a relativistic state is always constant, v≅1094 km/s. The characteristic energies of photons depend on the nature of the irradiated conductor and the cleanliness of its surface. The values of characteristic energies are determined by a simple formula, $h\nu = 3.4\ eV + \phi$. The process of transition of a particle from a relativistic state to a classical state and back is very similar to the latent thermal energy of aggregate transformations of matter. For an electron at $v_{max} = v_{min}$, the latent transition energy is zero:

$$Q_e = E_r - E_h = 0; \quad (7)$$

$$Q_e = \frac{m_e v^2}{2 * 1.6022 \cdot 10^{-19}} - 3{,}4\ eV = \frac{9.11 \cdot 10^{-31} \cdot (1.094)^2 \cdot 10^{12}}{2 * 1.6022 \cdot 10^{-19}} - 3.4\ eV = 0.$$

4. Sources of Information

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