Interaction of Relativistic Electrons with Photons

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Annotation

The total kinetic energy of a relativistic electron is composed of the kinetic energy of its translational motion and the relativistic energy of its proper rotation. Briefly about the main points.

Contents

Section 1. Total kinetic energy

The total kinetic energy of a relativistic electron is composed of the kinetic energy of translational motion and the relativistic energy of its own rotation:

$$E_k = \frac{mv^2}{2} + mc^2 - \frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}}\,; \qquad (1)$$

$$E_k = E_s + E_r\,.$$

The relativistic energy of an electron can be emitted as individual photons (spontaneous or bremsstrahlung). It can also participate in elastic collisions of electrons with external photons (direct and inverse Compton effects). The total kinetic energy of a particle changes in specific portions (quanta):

$$E_r = mc^2 - \frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}}\,. \qquad (2)$$

The process of energy quantization defines the energy levels of the electron states in a vacuum.

Section 2. Relativistic parameter

The total kinetic energy of an electron in neighboring states:

$$E_{k,i} = E_{s,i} + E_{r,i}; \qquad \text{— initial state.} \qquad (3)$$

$$E_{k,i+1} = E_{s,i+1} + E_{r,i+1}; \qquad \text{— neighboring state.}$$

Depending on the situation, the energy of a relativistic quantum is subtracted from or added to the total energy of the initial state. The electron mass is an invariant quantity. Elementary Relativity (ETR) uses a **relativistic parameter**—the ratio of the electron's velocity to the speed of light squared (the square of the relative velocity):

$$x_i = \frac{v_i^2}{c^2};\; x_{i+1} = \frac{v_{i+1}^2}{c^2}\;; \quad \text{— neighboring states.} \qquad (4)$$

The index $i -$ is the order of discrete values of the electron's effective velocity. The index runs through a series of natural numbers: $i = 0, 1, 2, 3, \dots n-1,\; n$. A value of zero corresponds to the maximum or minimum possible relativistic velocity of an electron in a vacuum.

Section 3. Bremsstrahlung

Let's consider the bremsstrahlung of photons from the wave surface of an ultra-relativistic electron. The relativistic energy is interpreted as the internal photon. The initial velocity of the electron is taken to be the speed of light, $v_0 = \mathrm{c}$.

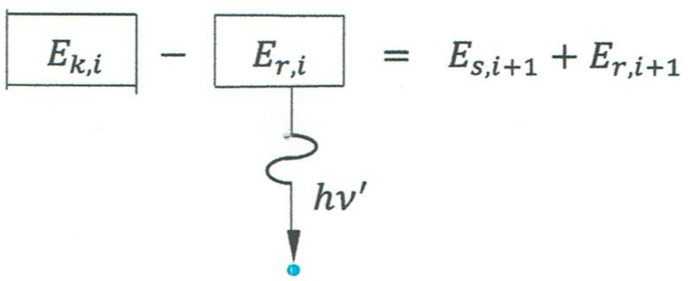

Figure 1

In Figure 1, ν' − is the frequency of the emitted photon, and h − is Planck's constant. After the emission of the internal photon, recombination of the total kinetic energy occurs. The remaining electron energy is again decomposed into translational and relativistic components according to the formula:

$$\frac{mv_i^2}{2} = \frac{mv_{i+1}^2}{2} + mc^2 - \frac{mc^2}{\sqrt{1+\frac{v_{i+1}^2}{c^2}}}; \qquad E_{r,i} = h\nu'. \qquad (5)$$

It should be noted that the movement of an electron close to the speed of light is not a metastable state for a material particle.

Section 4. Scattering of energetic photons

Let's consider the scattering of external energetic photons by relativistic electrons. The initial velocity of the electron is taken to be the minimum possible relativistic velocity, $v_0 = v_{min}$. This value is defined more precisely below in the text.

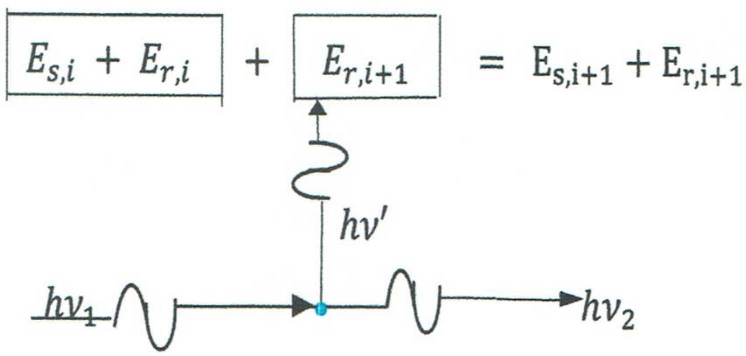

Figure 1

An electron receives a portion of relativistic energy, $E_{r,i+1}$, from an external photon. After the transfer of this energy quantum, the electron's total energy recombines according to the formula:

$$\frac{mv_{i+1}^2}{2} = \frac{mv_i^2}{2} + mc^2 - \frac{mc^2}{\sqrt{1+\frac{v_i^2}{c^2}}}; \qquad E_{r,i+1} = h\nu'. \tag{6}$$

The previous values of translational and relativistic energy are combined into a new value of translational energy, $E_{s,i+1}$. The order of the relative velocity indices is reversed.

Section 5. Recombination equations of relativistic parameters

The recombination of the total energy leads to two separate equations in terms of the relativistic parameters:

A) $(x_{i+1} - x_i + 2)^2(1 + x_{i+1}) - 4 = 0$, decrease in speed. (7)

B) $(x_i - x_{i+1} + 2)^2(1 + x_i) - 4 = 0$, increase in speed.

The axis of rotation of an electron in an atom undergoes precession. The precession energy is characterized by the fine-structure constant, α:

$$E_{h,i} = \frac{\alpha^2}{8}\frac{mc^2}{\sqrt{1+\frac{v_i^2}{c^2}}}.$$

When an electron transitions from a bound to a free state, the precession energy remains virtually unchanged. From the Fermi surface inside metals to the speed of light in a vacuum, the range of this energy variation is 3.4 to 2.4 eV. In the general equation for the total electron energy, precession enters as a constant with a negative sign. The exact equations for the recombination of relativistic parameters (x_i , x_{i+1}) are written as:

A) $(x_{i+1} - x_i + 2)^2(1 + x_{i+1}) - 4 = -\alpha^2 = (\alpha j)^2.$ (8)

B) $(x_i - x_{i+1} + 2)^2(1 + x_i) - 4 = -\alpha^2 = (\alpha j)^2.$

Solutions to the equations are sought relative to the parameter x_{i+1}. Precession is reactive energy, so the imaginary unit j is used. The constant α has no significant effect on energy recombination in a vacuum. However, taking it into account in type A, equations allows us to calculate the minimum relativistic velocity of an electron in a vacuum.

Section 6. Minimum relativistic speed

The minimum relativistic speed can be calculated by transforming the type A expression to the form:

$$x_{i+1}^3 + x_{i+1}^2(5 - 2x_i) + x_{i+1}(8 - 6x_i + x_i^2) + (x_i^2 - 4x_i + \alpha^2) = 0. \tag{9}$$

This expression forms a system of sequential equations at index i+1. Each solution to the preceding equation is the initial condition for the solution of the subsequent equation. The initial condition is taken to be $x_0 \approx 1$ (the speed of light). The equations have positive real solutions if the free term of expression (9) is less than or equal to zero, or greater than or equal to minus three. The remaining solutions are complex. A total of 16 equations, or 16 discrete values of the relativistic parameter, are formed.

It is not possible to use expression type A to determine the energy levels of an electron for the following reasons:

1) The motion of a material particle close to the speed of light is an unstable state; the maximum velocity is not precisely determined.

2) Sequential calculation of the squares of the relative velocity creates a systematic error that accumulates as the index $i + 1$ increases.

We find the speed of the electron at which the equality is fulfilled:

$$x_i^2 - 4x_i + \alpha^2 = 0\ ; \tag{10}$$

$$x_{min} = 2 - \sqrt{4 - \alpha^2}\ ;$$

$$v_{min} = c\sqrt{x_{min}} \cong \frac{1}{2}\alpha c = 1.09385 \cdot 10^6\ ms^{-1}.$$

To calculate the minimum relativistic speed, the exact value of the speed of light, $c = 299.792\ 458 \cdot 10^6\ ms^{-1}$, and the exact value of the fine-structure constant, $\alpha = 7.297\ 352\ 569 \cdot 10^{-3}$, are used.

Section 7. Quantization of kinetic energy

We consistently arrive at the need to analyze a type B equation. This expression also forms a system of successive equations of the form:

$$x_{i+1} = x_i + 2 - \frac{\sqrt{4-\alpha^2}}{\sqrt{1+x_i}}\ . \tag{11}$$

The initial value of the relativistic parameter is taken to be $x_0 = x_{min}$ from Section 6. The number of energy states of the particle is 16. We take the numerator of the fraction to be $\sqrt{4 - \alpha^2} \approx 2$. We obtain the equation in the form:

$$x_{i+1} = x_i + 2 - \frac{2}{\sqrt{1+x_i}}\ . \tag{12}$$

The expression for the fraction is expanded in a Taylor series. Since the initial value $x_0 \ll 1$, we only use the first two terms in the expansion:

$$\frac{1}{\sqrt{1+x_i}} \approx 1 - \frac{1}{2}x_i.$$

The equation can be written as:

$$x_{i+1} \approx x_i + 2 - 2 + x_i = 2x_i.$$

Then:

$$x_0 = 1 \cdot x_0 = 2^0 x_0 \ ;$$

$$x_1 = 2 \cdot x_0 = 2^1 x_0 \ ;$$

$$x_2 = 2 \cdot 2 \cdot x_0 = 2^2 x_0 \ ;$$

$$x_3 = 2 \cdot 2 \cdot 2 \cdot x_0 = 2^3 x_0 \ ;$$

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$$x_{16} = 2 \cdot 2 \cdot 2 \cdot \_\_\_\_ \cdot 2 \cdot x_0 = 2^{16} x_0 \ ;$$

We replace the sequence of discrete values of the relativistic parameter with the number **of quantum states** of the electron energy.

$$n = i + 1, \text{ где: } n = 0, 1, 2, 3, \dots \dots \dots 16. \qquad (13)$$

The value $i = -1$ should be understood as the classical state of the particle ($v < v_{min}$) preceding the relativistic state. The number of quantum states should equal the number of equations from Section 6. For metastable states, the relativistic parameter is determined by a power function:

$$x_n = 2^n x_0 \ . \qquad (14)$$

The resulting dependence works well at speeds $v \leq 10 \cdot 10^6 \ ms^{-1}$. At speeds greater than this value, the function begins to diverge from the calculated values using formula (12). A correction of the exponent is necessary for unstable conditions. In a scalar field of discrete speeds, a good approximation is provided by a power function of the form:

$$x_n = x_0 \cdot 2^n \cdot 2^{n \cdot \delta(n)} \ . \qquad (15)$$

Here $\delta(n) \ll 1$, the index responsible for the broadening of electron energy levels in a vacuum or the ranges of allowed energies. We assume that $\delta(n)$ is directly proportional to the number of the electron's quantum state. Then:

$$\delta(n) = k_n \cdot n + b \ .$$

The following values of k_n and b were obtained empirically:

$$k_n = 8.71471657 \cdot 10^{-4} \ ; \quad b = -16.42943314 \cdot 10^{-4}.$$

A consistent calculation of the relativistic parameter using formula (15) yields good agreement. For high-energy particles, strictly defined energy levels do not exist. Ranges of allowed energies are formed. The lower value of the range corresponds to the particle's metastable state. The upper value corresponds to an unstable state. Within these ranges, the emission and absorption of photon energy is possible without changing the particle's ground energy state. In all states, the total kinetic energy of an electron in a vacuum is calculated using the same formulas:

$$E_{s,n} = \frac{1}{2} mc^2 x_n; \tag{16}$$

$$E_{r,n} = mc^2 \left(1 - \frac{1}{\sqrt{1+x_n}}\right);$$

$$E_{h,n} = \frac{\alpha^2}{8} \frac{mc^2}{\sqrt{1+x_n}};$$

$$E_{k,n} = E_{s,n} + E_{r,n} - E_{h,n};$$

$$x'_n = x_0 \cdot 2^n \cdot 2^{n \cdot \delta(n)}; \quad - \quad \text{unstable condition;}$$

$$x_n = x_0 \cdot 2^n; \quad - \quad \text{metastable state;}$$

$$x_0 = \frac{\alpha^2}{4};$$

$$v_0 = \frac{1}{2} \alpha c;$$

$$v'_n = c\sqrt{x'_n}; \quad - \quad \text{unstable condition;}$$

$$v_n = c\sqrt{x_n}; \quad - \quad \text{metastable state.}$$

Section 8. Energy levels in a vacuum

It's impossible to show all the energy and velocity calculations in this article. However, the calculation of the energy level widths (ranges) is of interest.

$$\Delta E = E'_k - E_k .$$

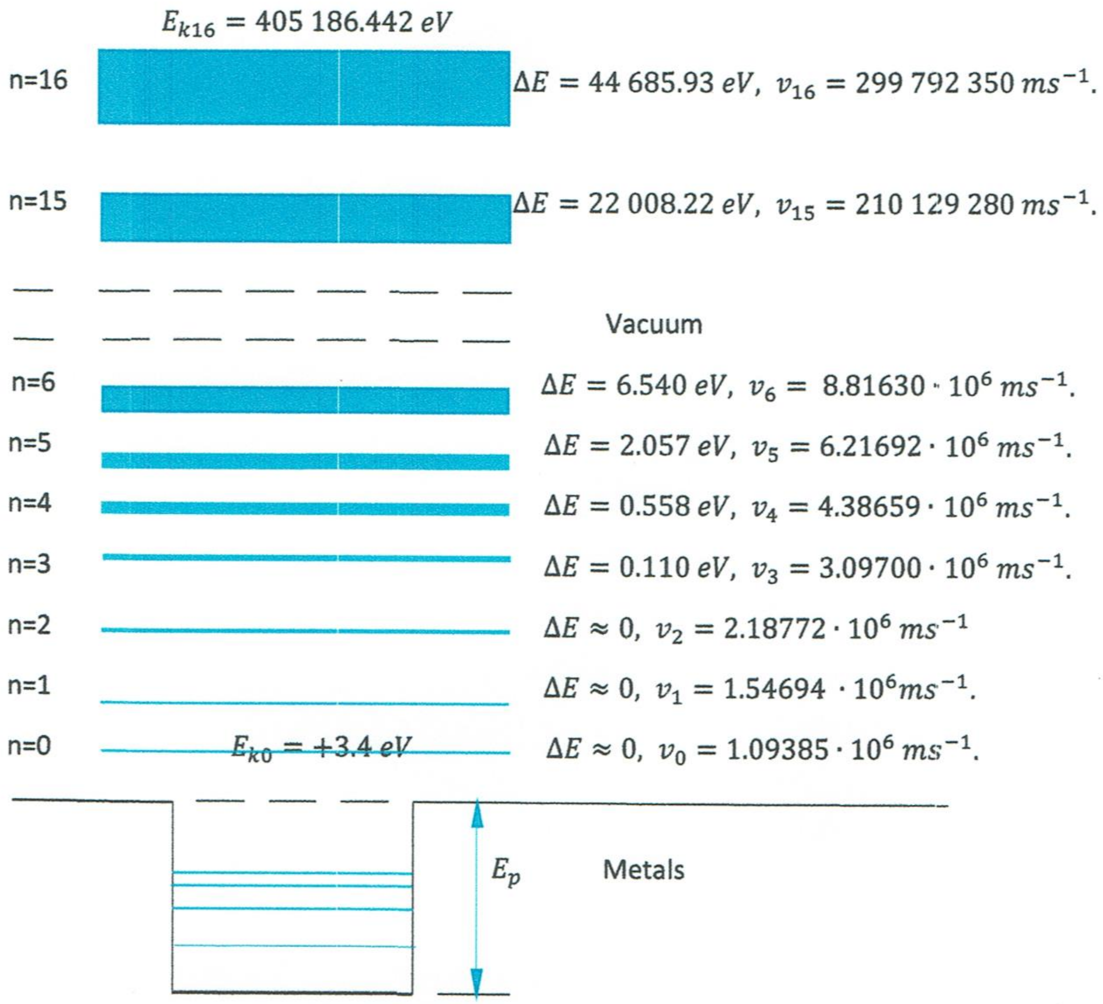
Here:

E_k – is the total kinetic energy of an electron in a **metastable** state;

E'_k – is the total kinetic energy of an electron in an **unstable** state.

The calculation results and energy level diagram are presented below.

n	$v' * 10^6\ ms^{-1}$	$v * 10^6\ ms^{-1}$	$E'_k\ eV$	$E_k\ eV$	$\Delta E\ eV$
0	1.09385	1.09385	03.402	03.402	≈ 0
1	1.54694	1.54694	10.204	10.204	≈ 0
2	2.18772	2.18770	23.811	23.812	≈ 0
3	3.09700	3.09387	51.133	51.023	00.110
4	4.38659	4.37540	106.000	105.442	00.558
5	6.21692	6.18775	216.325	214.268	02.057
6	8.81630	8.75080	438.408	431.868	06.540
7	12.51005	12.37550	885.877	866.868	19.014
8	17.76210	17.50160	1 788.117	1 736.020	52.097
9	25.23434	24.75100	3 607.689	3 471.027	136.662
10	35.87169	35.00320	7 273.933	6 927.956	345.977
11	51.02395	49.50200	14 642.634	13 669.288	973.346
12	72.62041	70.00640	29 353.598	27 317.738	2 035.86
13	103.42027	99.00400	58 341.903	53 638933	4 702.97
14	147.37199	140.01280	114 158.219	103 737.020	10 421.20
15	210.12928	198.00800	218 083.754	196 075.537	22 008.22
16	299.792 350	280.025600	405 186.442	360 500.517	44 685.93



In the diagram:

E_p − is the depth of the potential well for electrons in metals; It ranges from 3 to 10 eV. The surface of the potential well is taken as the reference point for the kinetic energy of **classical electrons** in a vacuum.

E_k − is the total kinetic energy of a **relativistic electron**; It is measured from an energy of +3.4 eV, which corresponds to the initial level of relativistic electron energies in a vacuum (n=0). The velocity values in the diagram are given for **unstable states**.

Section 9. Relationship of energy with electron momentum

Let's assume that a particle with an unknown structure acts on the electron's wave surface. This particle determines the relativistic energy and relativistic momentum ($E_{r,i}$; $p_{r,i}$). The electron's constant mass is concentrated in the Compton sphere. When the kinetic energy of the electron changes, the unknown particle transitions to the state of an internal photon.

$$E_r = mc^2 - \frac{mc^2}{\sqrt{1+\frac{v^2}{c^2}}} = h\nu'. \qquad (17)$$

Let us write down the basic equation of the relationship between energy and momentum for a relativistic particle [article from the book "VERSION of Elementary Theory of Relativity", Section 3.4].

$$E_{r,i}^2 = 2E_{r,i}mc^2 - p_{r,i}^2c^2.$$

We replace the relativistic energy of the particle with the equivalent photon, $E_{r,i}^2 = p_{\gamma,i}^2c^2$. The mass of the unknown particle is zero, m=0. We obtain the following relations:

$$p_{\gamma,i}^2 = -p_{r,i}^2\,;$$

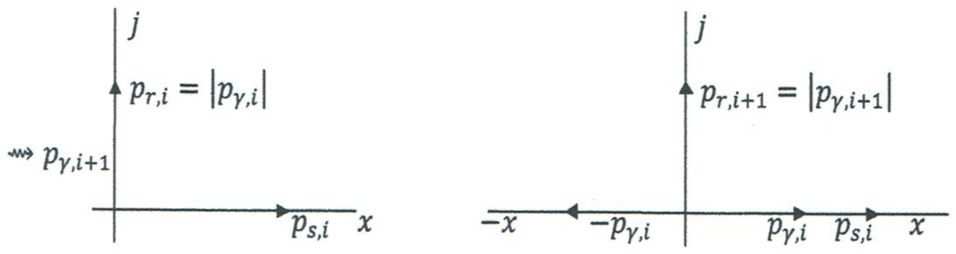
$$\vec{p}_{\gamma,i} = j\vec{p}_{r,i}\,;$$

$$|\vec{p}_{\gamma,i}| = |\vec{p}_{r,i}|.$$

The vectors of the relativistic momentum and the momentum of the internal photon are shifted in space at an angle of 90^0. They are equal in magnitude and **do not act simultaneously**. In the ideal case, the momentum of the internal photon at the moment of change in kinetic energy will be directed along the trajectory of motion, $\vec{p}_{\gamma,i}||\vec{p}_{s,i}$. Ideal cases are possible in the direct or inverse Compton effect. In non-ideal cases, the angle θ between the scattered photon and the trajectory of the electron must be taken into account [1].

Section 10. Electron and photon momenta

Let's consider an ideal case of particle momentum change. The electron's kinetic energy changes from the initial state to the final state. Below are the vector diagrams of momenta in adjacent states (in absolute units). Here, x denotes the coordinate axis along the electron's trajectory. We determine the electron's momentum along this trajectory.



a) Initial state

$p_{e,i} = p_{s,i}$

$p_{r,i} = |p_{\gamma,i}|$

b) Final state

$p_{e,i+1} = p_{s,i} \pm p_{\gamma,i} = p_{s,i+1}$

$p_{r,i+1} = |p_{\gamma,i+1}|$

In the final state, the vector $\vec{p}_{r,i}$ rotates clockwise or counter clockwise and becomes the vector $\vec{p}_{\gamma,i}$. The vectors $p_{r,i}, p_{r,i+1}$ – are the relativistic momentum vectors of the electron's proper rotation and act perpendicularly to the path of motion.

The plus sign corresponds to an increase in the electron's kinetic energy. A new quantum of relativistic energy is transferred from an external source (accelerating fields, energetic photons—the direct Compton effect). The electron absorbs energy in strictly defined portions. Momentum changes in discrete values.

The minus sign corresponds to a decrease in the electron's kinetic energy. A new quantum of relativistic energy is replaced by an internal source (the energy of proper translational motion $E_{s,i}$. The energy of the internal-photon is emitted in the bremsstrahlung field or transferred to low-energy photons—the inverse Compton effect. Momentum changes in discrete values.

Conclusions:

1. Discrete values of the relativistic parameter are independent of time and spatial coordinates. Only the square of the relative velocity ($x_i = \beta_i^2$) determines the quantum states of a particle in a vacuum. This situation is possibly related to the concept of locality and non-locality of physical processes in the surrounding space. Heisenberg's uncertainty principle is confirmed.
2. It's possible that the ranges of allowed energies of motion in a vacuum have additional sublevels. Mathematically, such a problem is solved quite simply, based on expression (15). However, no logical justification is apparent. The number of cubic equations based on expression (9) does not exceed 16.

3. The electron's transition to a relativistic state occurs abruptly, with a minimum velocity of $1.09385\ ms^{-1}$. A possible cause is the actual effect of the physical vacuum.
4. The electron energy equations do not require renormalization. The total kinetic energy does not tend to infinity upon reaching the speed of light.

Sources

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