The Bohr Magneton and the Anomalous Magnetic Moment

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ABSTRACT

This article is based on a spherical model of a relativistic electron. The model consists of an outer wave sphere and an inner Compton sphere.

Keywords: electron; velocity; magnetic moment; charge; fine structure constant.

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1. Introduction

This article is based on a spherical model of a relativistic electron. The model consists of an outer wave sphere and an inner Compton sphere. Standing electromagnetic waves of relativistic energy pulsate between the spheres. For such waves to exist, reflection boundaries in the form of charged surfaces are necessary. We use the results of Section A.6 of the publication [1] on the author's website: "On the Structure of the Relativistic Electron."

2. Reflection Boundaries

Suppose that a small positive charge $+q_1$ is located on the surface of the Compton sphere (Figure 1). The sum of the charges of both spheres is equal to the charge of the electron, $q_1 - q_2 = -e$. We find the electric potential of each sphere separately.

Fig. 1

Potential of the first sphere,

$$\varphi_1 = k\frac{q_1}{R_1} - k\frac{q_2}{R_2}.$$

The potential of the second sphere,

$$\varphi_2 = k\frac{q_1}{R_2} - k\frac{q_2}{R_2}.$$

The potential difference,

$$\Delta\varphi = \varphi_1 - \varphi_2 = k\frac{q_1}{R_1} - k\frac{q_2}{R_2} - k\frac{q_1}{R_2} + k\frac{q_2}{R_2}.$$

After simple transformations, we obtain,

$$\Delta\varphi = k\frac{\left(\frac{q_1}{q_2} - \frac{R_1}{R_2}\right)}{R_1 q_2} + k\frac{q_2 - q_1}{R_2}. \tag{1}$$

Let's make the assumption: **the absolute value of the potential difference between the Compton sphere and the outer sphere of the electron is equal to the potential difference between a free electron and the zero potential at infinity.**

Then the first term in formula (1) is zero. The following identities are satisfied:

$$\frac{q_1}{q_2} \equiv \frac{R_1}{R_2} \quad и \quad q_1 - q_2 \equiv -e\,. \tag{2}$$

For the radius of the electron's outer sphere, we take the radius of the first Bohr orbit in the hydrogen atom, $r_n = R_2 = 52.91772 \cdot 10^{-3}\ nm$. The radius of the Compton sphere, , $r_c = R_1 = \Lambda/2\pi$. Then the charge ratio is exactly equal to the fine structure constant α:

$$\frac{q_1}{q_2} = \frac{R_1}{R_2} = \frac{r_c}{r_n} = \frac{\Lambda}{2\pi R_2} = \frac{2.42631*10^{-3}nm}{2\pi*52.91772*10^{-3}nm} = 0.0072973525646 = \alpha.$$

If we take $q_2 - q_1 = e = 1.60218 \cdot 10^{-19}\ K$, as a first approximation, then each of the charges is equal to: $q_1^+ \cong +0.01178 \cdot 10^{-19}K$; $q_2^- \cong -1.61396 \cdot 10^{-19}K$. This means that the boundaries of reflection of standing waves can be taken to be the positively charged Compton sphere and the negatively charged sphere of the electron. For other values of the radius R_2, the system of equations is solved based on identities (2). The absurdity of expression (1) with the numerical equality of the radii and charges of the spheres suggests that a **free relativistic electron** is a body of rotation of the "electron" disk and **can take a ring shape only in a bound state**. The ratio of the speed of an electron in the first orbit of a hydrogen atom to the speed of light is taken as the fine-structure constant. Arguing with this statement is futile; it must be accepted as fact. The electron's axis of rotation is directed perpendicular to the trajectory of its translational motion.

3. Kinematics of charge motion

Let us concentrate a negative charge q_2 at a single point on the equator of the electron's outer surface. The charge is rigidly bound to the wave surface. The points on the equator rotate with velocity v_2. The sphere rolls through space as if on a flat plane without slipping. To a first approximation, the rotational velocity is equal to the translational velocity and corresponds to the first orbit of the Bohr hydrogen atom, $v_2 = \alpha \cdot c \cong 2.1877 \cdot 10^3\ km \cdot s^{-1}$. The ratio of this velocity to the speed of light is taken as the fine-structure constant α. The charges move in a single plane along the equator of the spheres. In the International System of Units (SI), the fine-structure constant is defined as follows:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \quad \Longrightarrow \quad v_2 = \alpha \cdot c\ .$$

On the surface of the Compton sphere, a positive charge q_1 rotates at the speed of light, $v_1 = c$.

The proposed kinematic scheme is a greatly simplified model of the electron, but it allows one to directly calculate the intrinsic magnetic moment of a charged particle.

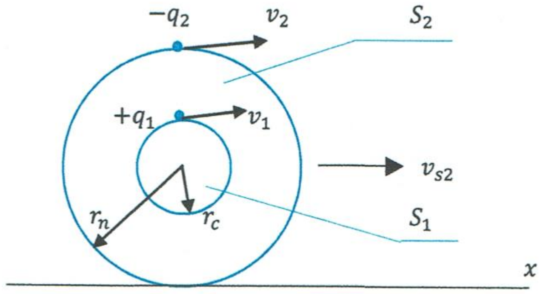

Fig. 2

Here:

v_2 – is the rotational velocity of charge q_2^-;

v_1 – is the rotational velocity of charge q_1^+;

v_{s2} – is the translational velocity of the electron;

r_c – is the radius of the Compton sphere;

r_n – is the radius of the wave sphere;

S_1 – is the circulation area for charge q_1;

S_2 – is the circulation area for charge q_2.

4. Basic relationships of the model

Equations for charges:

$$q_1 - q_2 = -e \qquad q_2 = q_1 + e\,;$$

$$\frac{|q_1|}{|q_2|} = \alpha \qquad q_1 = q_2 \cdot \alpha\,.$$

Effective values of charges:

$$q_2 = \frac{e}{1-\alpha}\,; \qquad q_1 = \frac{\alpha \cdot e}{1-\alpha}\,.$$

Ratio of parameters:

$$\frac{q_1}{q_2} = \frac{r_c}{r_n} = \frac{v_2}{v_1} = \alpha\,;$$

$$v_2 = \alpha c\,; \quad v_1 = c\,;$$

$$r_n = \frac{r_c}{\alpha}\,.$$

Areas of charge circulation in a closed circuit:

$$S_2 = \pi \cdot r_n^2\,;$$

$$S_1 = \pi \cdot r_c^2\,.$$

Compton sphere radius:

$$r_c = \frac{\Lambda}{2\pi}; \quad \Lambda = \frac{\mathrm{h}}{m_e \cdot c} \Longrightarrow r_c = \frac{h}{2\pi m_e}.$$

Here: $m_e -$ is the rest mass of the electron; $h -$ is Planck's constant.

Intrinsic magnetic moments:

$$p_{m2} = \frac{q_2^- \cdot v_2 \cdot S_2}{2\pi r_n}; \qquad p_{m1} = \frac{q_1^+ v_1 S_1}{2\pi r_c}.$$

We substitute the current parameters into the magnetic moment formulas. **The charges have opposite signs.** We assume p_{m2} has a plus sign and p_{m1} has a minus sign. Then the electron's intrinsic moment is:

$$p_{me} = p_{m2} - p_{m1} = \frac{1}{2}\left(\frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}\right)\frac{eh}{2\pi m_e} = \frac{1}{2}\frac{eh}{2\pi m_e}. \qquad (3)$$

Dirac introduces his own constant of angular momentum in his equations:

$\hbar = \frac{h}{2\pi}$; (Dirac constant).

Expression (3) is simplified:

$$p_{me} = \frac{e\hbar}{2m_e} = \mu_B. \qquad (4)$$

The last expression is called the Bohr magneton in electrodynamics. This moment is **independent of the electron's trajectory and external fields.** The Stern-Gerlach experiments showed that the electron's actual magnetic moment is twice as large as the theoretical one. Quantum electrodynamics resolved this situation quite simply, writing this expression as follows:

$$\vec{p}_{em} = g_e \cdot \frac{e\hbar}{2m_e} \cdot \vec{s}. \qquad (5)$$

$g_e -$ is the electron factor ($g_e \approx 2$).

$s -$ is the spin quantum number, $s = \pm\frac{1}{2}$.

The factor – g_e compensates for the 2 in the denominator, while the spin indicates the direction of the magnetic moment and is an intrinsic characteristic of the electron.

5. Anomalous magnetic moment

Subsequent quantum electrodynamic calculations (Feynman diagrams) and experimental data showed that the $g_e -$ factor is slightly greater than 2. It is customary to write this parameter as:

$$g_e = 2(1 + a_e).$$

Here, a_e is called the electron's anomalous magnetic moment. With each charge rotation, the Bohr magneton must fit into the plane of the electron disk at constant values. Let's identify an **elementary sector** on the circle with a small central angle $\boldsymbol{\alpha}$ (the fine structure constant). Let's determine the total number of such sectors on the circular disk:

$$n = \frac{2\pi}{\alpha} = 861.022576552.$$

Sectors can only be quantized as integers. Taking 861 as a quantum number is not possible. A spatial gap between the first and last sectors on the circle is impossible. The Bohr magneton can be quantized as fractions of $\boldsymbol{\alpha}$ only by adding **the area of another elementary sector.** The first and the added sectors will partially overlap. The reference point of the sectors on the circle will constantly shift as the electron rotates.

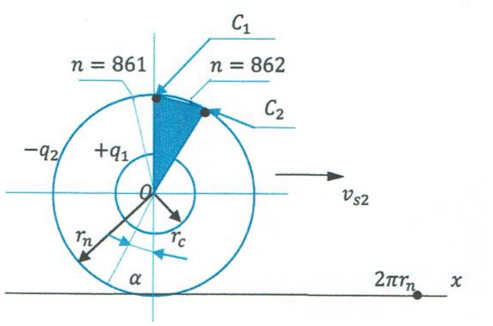

Fig. 3

Then, the area of sector C_1OC_2 must be taken into account twice when calculating the magnetic moment. **The origin of the sectors on circle $\boldsymbol{C_1}$** will shift in space and time with each revolution to position C_2, and so on until it returns to its original position. This shifting process will repeat itself over and over again. The question arises: why does this happen? Perhaps the disk's geometry is not circular, perhaps the disk is slipping relative to its trajectory—anything is possible. The author does not know the answer to this question, and does not wish to speculate. Let's proceed from the facts.

Let us divide the magnetic moment of the electron into $\boldsymbol{n}$ elementary parts and add the resulting value to the whole Bohr magneton:

$$\mu'_B = \mu_B + \frac{\mu_B}{n}; \qquad (6)$$

$$\vec{p}_{me} = 2(1 + \frac{\alpha}{2\pi})\mu_B \cdot \vec{s}.$$

The expression $\frac{\alpha}{2\pi}$ is the first Schwinger correction. The area of the added sector must be equal to the area of the elementary sector with central angle $\boldsymbol{\alpha}$. This requirement leads **to deformation of all sides of the added sector**. The true position of point C_2 on the outer circumference of the disk must be found. The spherical electron model and the mathematics involved become significantly more complex.

Without further proof or explanation, the author presents the basic formula for the electron's anomalous magnetic moment.

As a first approximation:

$$a_e = \frac{\alpha}{2\pi}(1-\alpha) + \frac{\alpha^2}{8}(1+\alpha)\,; \qquad (7)$$

In the second approximation:

$$a_e = \frac{\alpha}{2\pi}(1-\alpha) + \frac{\alpha^2}{7{,}985}(1+\alpha)\,. \qquad (8)$$

The calculation accuracy for $\boldsymbol{a_e}$ is seven decimal places.

In the third approximation:

The author turned to Google AI to expand the denominator 8 into powers of α. The final equation ($\alpha = 7.2973525611 \cdot 10^{-3}$):

$$a_e = \frac{\alpha}{2\pi}(1-\alpha) + \frac{\alpha^2}{8-2\alpha-9{,}3\alpha^2}(1+\alpha) = 0.00115965218047\,.$$

Before us is the balance equation: at a small angle $\boldsymbol{\alpha}$, the compression geometry of the added sector forces the convexity of the arc to compensate for the loss of the sector's area, tending to return the sum to the original ideal fraction α/2π .

Explanation:

■ $\frac{\alpha}{2\pi}(1-\alpha) -$ Lateral compression of the added sector's area (truncation), which narrows slightly with decreasing coefficient (1-α).

■ $\frac{\alpha^2}{8}(1+\alpha) -$ Expansion of the gap (convexity of the sector's arc) or deflection of the arc above the chord, which increases with the coefficient (1+α).

■ Mechanical interpretation. Imagine that a soft, plastic circular sector is compressed from the sides. Its angular fraction decreases, which yields the first term with a minus sign (1-α). According to the laws of geometry and mechanics, the displaced volume is pushed outward, and the sector's arc deflects more strongly, yielding the second term with a positive sign (1+α).

6. Conclusion

In this case, the physical model of the electron (with a limited number of parameters) yields a value for the intrinsic magnetic moment no worse than quantum electrodynamics.

Sources of information

1. Michael Haletsky, "On the structure of the relativistic electron," 2022, https://halmich.ru , publications .

2. Anomalous magnetic moment of the electron https://ru.wikipedia.org/wiki/%D0%90%D0%BD%D0%BE%D0%BC%D0%B0%D0%BB%D1%8C%D0%BD%D1%8B%D0%B9_%D0%BC%D0%B0%D0%B3%D0%BD%D0%B8%D1%82%D0%BD%D1%8B%D0%B9_%D0%BC%D0%BE%D0%BC%D0%B5%D0%BD%D1%82